Appendix: Problems

2 RC Filter

Problem 2.1 Calculate the output signal of the RC filter for a sinusoidal input signal $A_i \sin(\omega_0 t)$. Make use of Euler's formula ($\sin y = (e^{jy} - e^{-jy})/2j$) and $T(j\omega) = \frac{1}{\sqrt{1 + (RC\omega)^2}} e^{j\Phi(\omega)}$ with $\Phi(\omega) = \operatorname{atan}(-\omega RC) = -\operatorname{atan}(\omega RC)$.

Problem 2.2 Determine graphically the modulus of the frequency response function for a RC filter with $R = 4{,}0\Omega$ and $C = 1{,}25F/2\pi = 0{,}1989495F$ ($1\Omega = 1(V/A)$, $1F = 1A\sec/V$). Where is the pole position in the s-plane? For the plot use frequencies between 0 and 5 Hz.

Problem 2.3 Calculate the frequency response of the RC filter from Problem 2.2 using the Digital Seismology Tutor (DST).

Problem 2.4 Let us end this chapter by considering an example directly related to our daily life. Consider a savings account with a monthly interest rate of α percent. The money which is deposited at time $t = nT$ is supposed to be $x(nT)$. $y(nT)$ represents the money in the account at time nT (before the deposit of $x(nT)$ is made), and $y(nT + T)$ is the money one sample (1 month) later. Determine the difference- and differential equations of the system using the forward difference ($\dot{y}(t) \approx \frac{y(nT + T) - y(nT)}{T}$). Start out with the balance at time $t = nT + T$ which can be written as $y(nT + T) = y(nT) + \alpha y(nT) + \alpha x(nT) + x(nT)$. Calculate the transfer function using Laplace transform (use equation (2.26)). Is the system stable? Could we use an RC filter to simulate the savings account?

3 General linear time invariant systems

Problem 3.1 System with two poles. Consider three different cases. a) Put both poles at (-1.2566, 0). b) Put one pole at location (-1.2566, 0) and the other one at (1.2566, 0). c) Put both poles at (1.2566, 0). For the input signal, use a spike at the center position of the window (for DST an internal sampling frequency of 100 Hz and a window length of 2048 points works well). What types of impulse response functions do you expect in each case? Will the frequency response functions be different? What changes do you expect for the frequency response functions with respect to the system in Problem 2.3?

Problem 3.2 Use the argument given above to determine the frequency response for Problem 2.3 if you add a zero at position (1.2566, 0)?

Problem 3.3 How can the following two statements be proven for a general LTI system? a) If a system is minimum phase it will always have a stable and causal inverse filter. b) Any mixed phase system can be seen as a convolution of a minimum phase system and a filter which only changes the phase response but leaves the amplitude response as is (all-pass filter).

Problem 3.4 How can we change the two-sided impulse response from Problem 3.1b into a right-sided one without changing the amplitude response? Keyword: allpass filter.

Problem 3.5 Consider a system with a pole and a zero on the real axis of the s-plane. Let the pole position be (-6.28318, 0), and the zero position (.628318, 0). What is the contribution of the zero to the frequency response function?

Problem 3.6 Move the pole position of a double pole at (-1.2566, 0) in steps of 15^o (up to 75^o, and 85^o) along a circle centered at the origin and passing through the original double pole. Calculate the impulse response functions and the amplitude portions of the frequency response functions.

Problem 3.7 Use the pole-zero approach to design a notch filter suppressing unwanted frequencies at 6.25 Hz. What kind of singularities do we need? How can we make use of the result of Problem 3.6?

Problem 3.8 From the shape of the frequency response function in Fig. 3.4, determine the poles and zeros of the corresponding transfer function.

4 The seismometer

Problem 4.1 Most seismometers operate on the principle of a moving coil within a magnetic field. Hence, they do not record the ground displacement but the ground velocity. Are equations (4.27) - (4.30) also valid for this kind of systems?

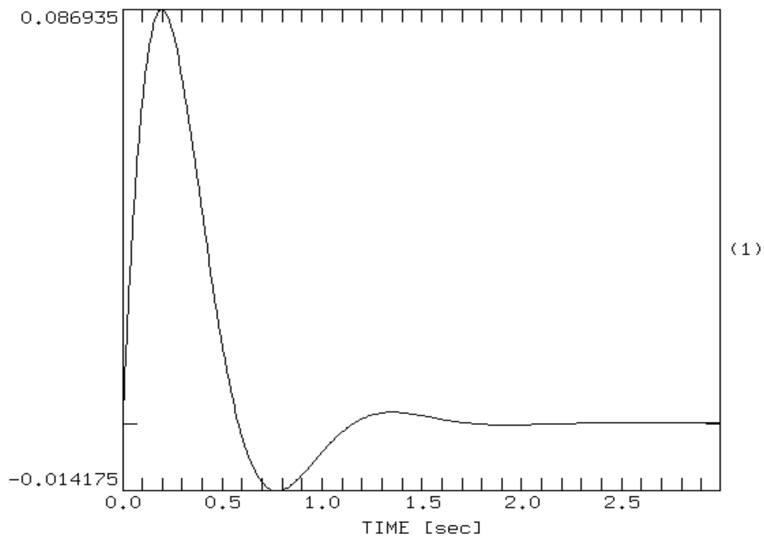

Fig. 4.1 Seismometer calibration pulse (response of an electrodynamic seismometer to a step function in acceleration)

Problem 4.2 The calibration signal shown in Fig. 4.1 is the response of a "velocity proportional sensor" to a step function in acceleration. Read the first two peak amplitude values a_k and the damped period from Fig. 4.1. Determine the damping constant h and the natural frequency f_0 of the system.

Problem 4.3 What is the theoretical relationship between the response of a "velocity proportional sensor" to a step function in acceleration as shown in Fig. 4.1 and the displacement impulse response?

Problem 4.4 Calculate the impulse response (and spectrum) for a displacement seismometer with an eigenfrequency of 1 Hz and damping factors of $h = 0{,}25, 0{,}5, 0{,}62$, respectively. How do the locations of the poles change for the different damping constants?

Problem 4.5 How do we have to change the pole and zero distribution if we want to change the seismometer from Problem 4.4 into an electrodynamic system recording ground velocity?

5 The sampling process

Problem 5.1 What is the general relationship between the discretization frequency, the signal frequency of a sinusoidal input signal ("input frequency") and the dominant frequency of the reconstructed signal ("output frequency")? Use DST to generate sinusoidal signals for an internal sampling frequency of 1024 Hz, a window length of 2048 points, and signal frequencies from 1 - 20 Hz in steps of 1 Hz. Discretize and reconstruct each signal using a discretization frequency of 10 Hz and note the dominant frequency and the maximum amplitude of the reconstructed signal. You can determine the dominant frequency of a signal in Hz easily by measuring the dominant signal period on the DST screen in seconds and taking the reciprocal value. From the table of input frequencies, output frequencies and output amplitudes, try to infer the rule for calculating the output frequency for a given signal frequency and a given digitization frequency. Hint: The so called Nyquist frequency (half of the discretization frequency) is also referred to as the *folding frequency*. Think of the "frequency band" as a foldable band which is folded at multiples of the Nyquist frequency. Mark the corresponding pairs of (input frequency, alias frequency) on this band. It may help to actually cut out a paper band and folding it.

Problem 5.2 What is the highest frequency which can be reconstructed correctly using a 'discretization frequency' of 10 Hz?

Problem 5.3 What would be the alias frequency for an input signal of 18.5 Hz and a discretization frequency of 10 Hz?

6 Analog-to-digital conversion

Problem 6.1 The Wood-Anderson magnitude is defined as $M_{WA} = \log_{10}(A) - \log_{10}(A_0)$ with A being the amplitude [in mm] measured on a Wood-Anderson displacement instrument and $-\log_{10}(A_0)$ being the distance correction which is exactly 3 for 100 km. What is the required dynamic range of a digital Wood-Anderson-equivalent instrument to record both a magnitude 0 and a magnitude 6 earthquake in 100 km distance "on scale"? Hint: Consider the theoretical Wood-Anderson trace amplitudes for both earthquakes in 100 km distance.

Problem 6.2 What are the differences in dynamic range (in dB) between a plain 16 bit ADC and a gain ranging ADC with 8 bit for the gain and 8 bit for the mantissa?

Problem 6.3 What happens during delta modulation for a rapidly rising input signal? What happens if the input signal is constant? For this demonstration with DST, generate a centered spike function of spike amplitude 100 using an internal sampling frequency of 100 Hz and a window length of 512 points. Integrate the resulting signal to obtain a centered step function of amplitude 1, which contains both a rapidly rising and two constant parts. Apply the delta modulation simulation within DST using a LSB value of 0.01 and discuss the differences between actual input signal and predicted input signal.

Problem 6.4 Perform the two types of delta modulation discussed above on a sinusoidal test signal. For this demonstration, set the internal sampling frequency in DST to 256Hz and the window length to 512 points (*Modify* option of the *Setup* menu). Next, generate a sinusoidal input signal using the *Test Signals -> Sine/Cosine* option of the *Traces* menu. For both the signal amplitude and the signal frequency enter a value of 1.0. First, simulate plain delta modulation using a LSB value of 0.05. Next, compare the resulting demodulated signal with the one you obtain if the integration is performed before delta modulation. In the latter case, adjust the LSB value for the quantization to the maximum peak to peak amplitude of the signal to be fed to the delta modulator. For the lowpass filter use a Butterworth filter (forward/backwards) using a single section and a corner frequency of 4 Hz.

Problem 6.5 Use the DST to simulate sigma-delta analog-to-digital conversion on a 2 second long sinusoidal test signal of peak amplitude 1 and a signal frequency of 1 Hz. For the internal sampling frequency use 5120 Hz. Decimate the output signal of the sigma-delta modulator in 2 decimation stages. Use a decimation ratio of 16 for the first step followed by a decimation ratio of 4. Try to find the optimum LSB value.

7 From infinitely continuous to finite discrete

Problem 7.1 Given a finite sequence of numbers $x[n]$ ($n = 0$ to $N-1$), which was obtained by sampling of a infinite continuous-time signal $x(t)$ (sampling interval T sec), what is the relationship between the DFT given in the notation of (7.29) and the Fourier transform given by (7.6) provided a) that the sampling was done in accordance with the sampling theorem, and b) that the amplitude of the signal $x(t)$ is zero outside of the finite time window ($0 \le t < NT$)? Hint: Use the rectangular rule to approximate the Fourier transform integral and compare the resulting finite sum for radian frequencies $\omega_k = k \cdot \frac{2\pi}{TN}$ to the expression for the DFT.

Problem 7.2 Which type of impulse response function would result if we want to use the inverse Fourier transform for evaluation of a system containing two poles on the real axis as shown in Fig. 7.5?

Problem 7.3 How do time shifts effect the region of convergence of the z-transform? Argue by using the shifting theorem for z-transforms ($x[n - n_0] \Leftrightarrow z^{-n_0}X(z)$) for positive and negative n_0.

Problem 7.4 How many samples long should the zero padding be in the example above in order to eliminate the wrap around effect?

8 The digital anti-alias filter

Problem 8.1 Do roots on the unit circles belong to the minimum or the maximum phase part of the impulse response? Start by considering a short 3 point wavelet with roots exactly on the unit circle. Why can unit circle roots be ignored for the purpose of removing the acausal filter response from seismic records?

Problem 8.2 Demonstrate the performance of the removal of the maximum phase component of the FIR filter response with real data using DST. Data traces can be loaded into DST using the *File -> Open File with Data Trace* option from the main menu. The subdirectory *FIR20HZ* contains data examples sampled at 20 Hz with visible precursory FIR filter effects prior to the P-wave onset (e. g. file example20HZ). Use option *FIR2CAUS* in the *Utilities* menu with the correction filter file quant20Hz.prt.

9 Inverse and simulation filtering of digital seismograms

Problem 9.1 For which frequencies do you expect regularity problems for a digital recording system?

Problem 9.2 Discuss the stability and phase properties of the terms $1/T_{act}^{min}(z)$ and $1/T_{act}^{max}(z)$ in (9.7).

Problem 9.3 Use the bilinear transformation with frequency warping to design a recursive filter to simulate a 'displacement' seismometer with a seismometer eigenfrequency of 1/120 Hz, a damping factor of 0.707, and a sampling frequency of 20 Hz.

Problem 9.4 Determine the difference equation for a deconvolution filter which completely removes the instrument defined in Problem 9.3.

Problem 9.5 Determine the difference equation for a deconvolution filter for an electrodynamic sensor with the same eigenfrequency, damping and sampling frequency as in Problem 9.3.

10 The measurement of wavelet parameters from digital seismograms

Problem 10.1 Fig. 10.3 shows three signals with different signal character and frequency content. The corresponding spectra are shown in Fig. 10.4. Discuss qualitatively what happens to these signals if they are recorded with the instrument for which the amplitude frequency response function is shown in Fig. 10.5. Under which circumstances would it be justified to call this a ‘velocity recording system’?

Problem 10.2 What are the proper units for a displacement-, a velocity-, and an acceleration frequency response function of a digital recording system (including seismometer)?

Problem 10.3 Given the velocity impulse response function for a seismic recording system, how can one calculate the corresponding displacement- and acceleration impulse response functions in both time- and frequency domain?

Problem 10.4 Determine the displacement frequency response function for an electrodynamic seismometer system with eigenfrequency $f_0 = 0{,}0082706$ Hz, $h = 0{,}718$ and a generator constant of $G = 1500$ [V/m/s]. Use equation (10.7) and convert the resulting transfer function into the required form. In addition, express the frequency response function by the roots of the transfer function.

Problem 10.5 Calculate the displacement frequency response function relating ground motion [nm] to [counts] for the seismometer in Problem 10.4 given an LSB of 2.5 μV.

Problem 10.6 In order to practice the conversion of frequency response functions one more time, convert the displacement frequency response function from Problem 10.5 into the corresponding velocity response function. Discuss the various possible approaches.

Problem 10.7 For a displacement frequency response function $T_{disp}(j\omega) = C_{disp} \cdot F_{disp}(j\omega)$, calculate the scale factor C_{disp} in (10.11) from the knowledge of g_d and the calibration period T_{cal}.

Problem 10.8 Given the displacement calibration gain, g_d, at the calibration period T_{cal}, derive the relationships between g_a, g_v, and g_d, and give the scale factor C_{disp} in (10.11) in terms of one of these quantities.

Problem 10.9 Imagine a sensor for which the output voltage is proportional to ground velocity for frequencies above 1 Hz. The sensor output (generator constant 100 V/m/s) is amplified by a factor of 250 before being fed into a 20 bit ADC with an LSB of 1 μV. Calculate the values of g_v and g_d for calibration frequencies of 5 and 10 Hz, respectively. Calculate the velocity- and displacement frequency response functions for a damping factor of 0.7. Where do g_v and g_d plot in these figures?

Problem 10.10 Imagine the following signal has been recorded on the instrument defined in Problem 10.9. What would be corresponding peak-to-peak amplitude of the ground displacement in nm?

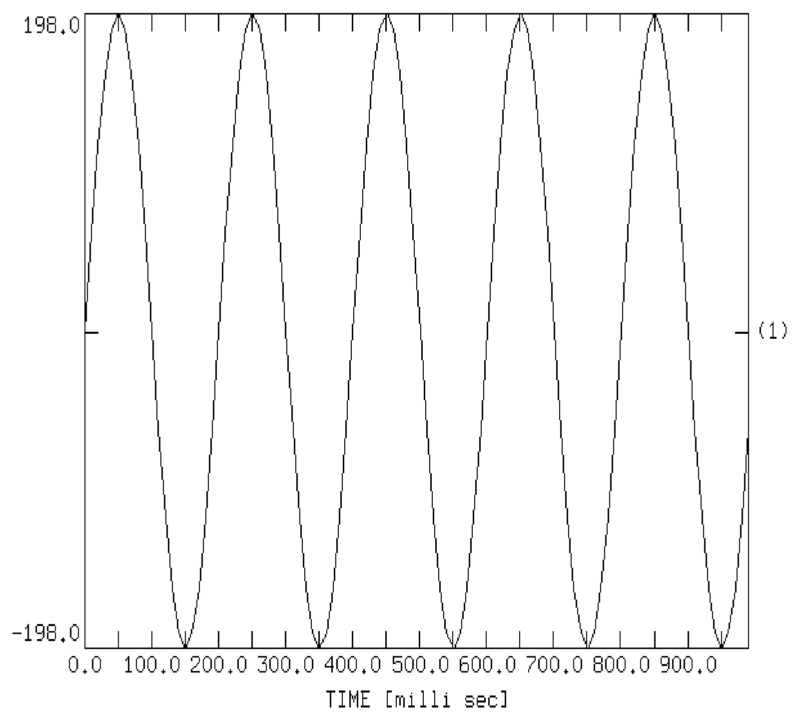

Fig. 10.1 Signal recorded with the instrument defined in Problem 10.9.

Problem 10.11 Can the implementation of the WWSSN SP seismometer by bilinear transform preserve the phase properties of the analog system? Argue from the fact that the resulting recursive filter has a causal impulse response function.

Problem 10.12 Calculate the rise time from the minimum to the maximum amplitude for a sinusoidal signal $A_0 \cdot \sin(\omega t)$.

Problem 10.13 Calculate the impulse and step response functions for a causal and an acausal Butterworth LP filter with 8 poles and a corner frequency of 1 Hz using DST. Discuss time delays and the minimum detectable pulse duration and rise times for potential filter signals.