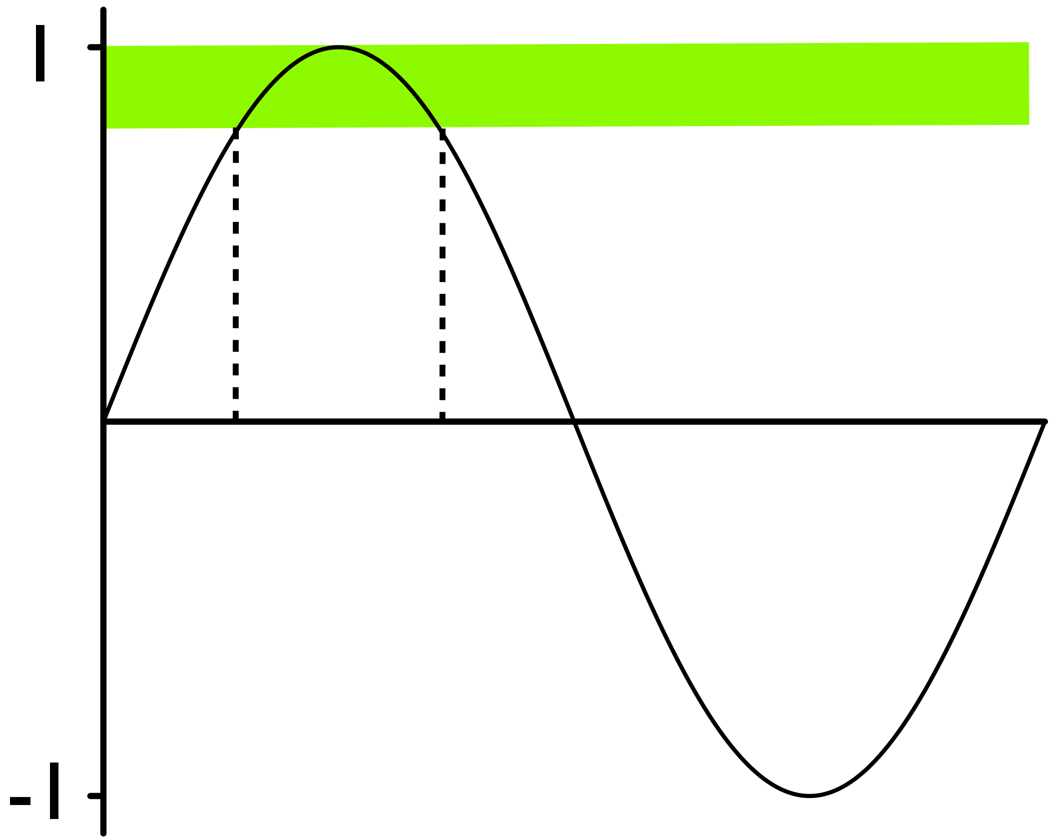
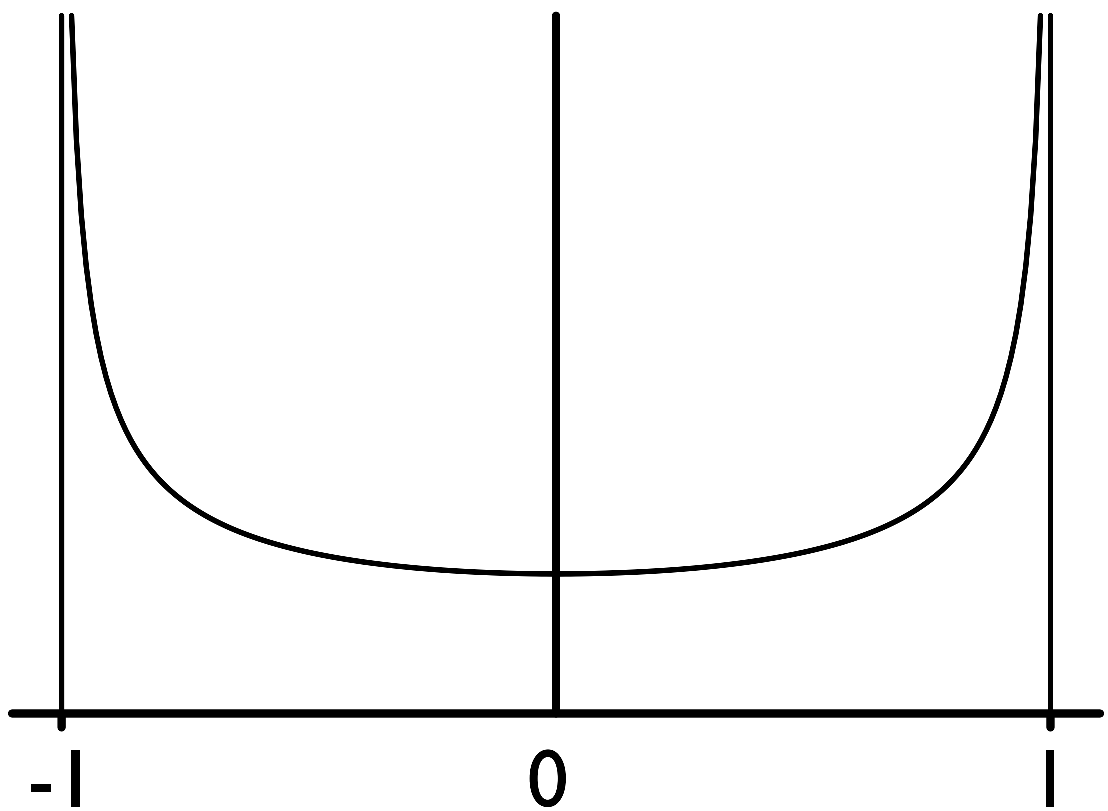
7. Using noise for tomography

Noise, cross correlation, phase velocities, and tomography

a sine wave



probability density function for a sine wave

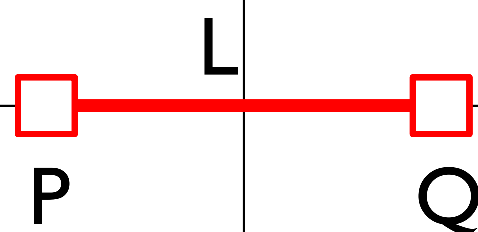


Starting point:

Much of the “noise” recorded at a station is fundamental mode Rayleigh and Love waves arriving from different directions

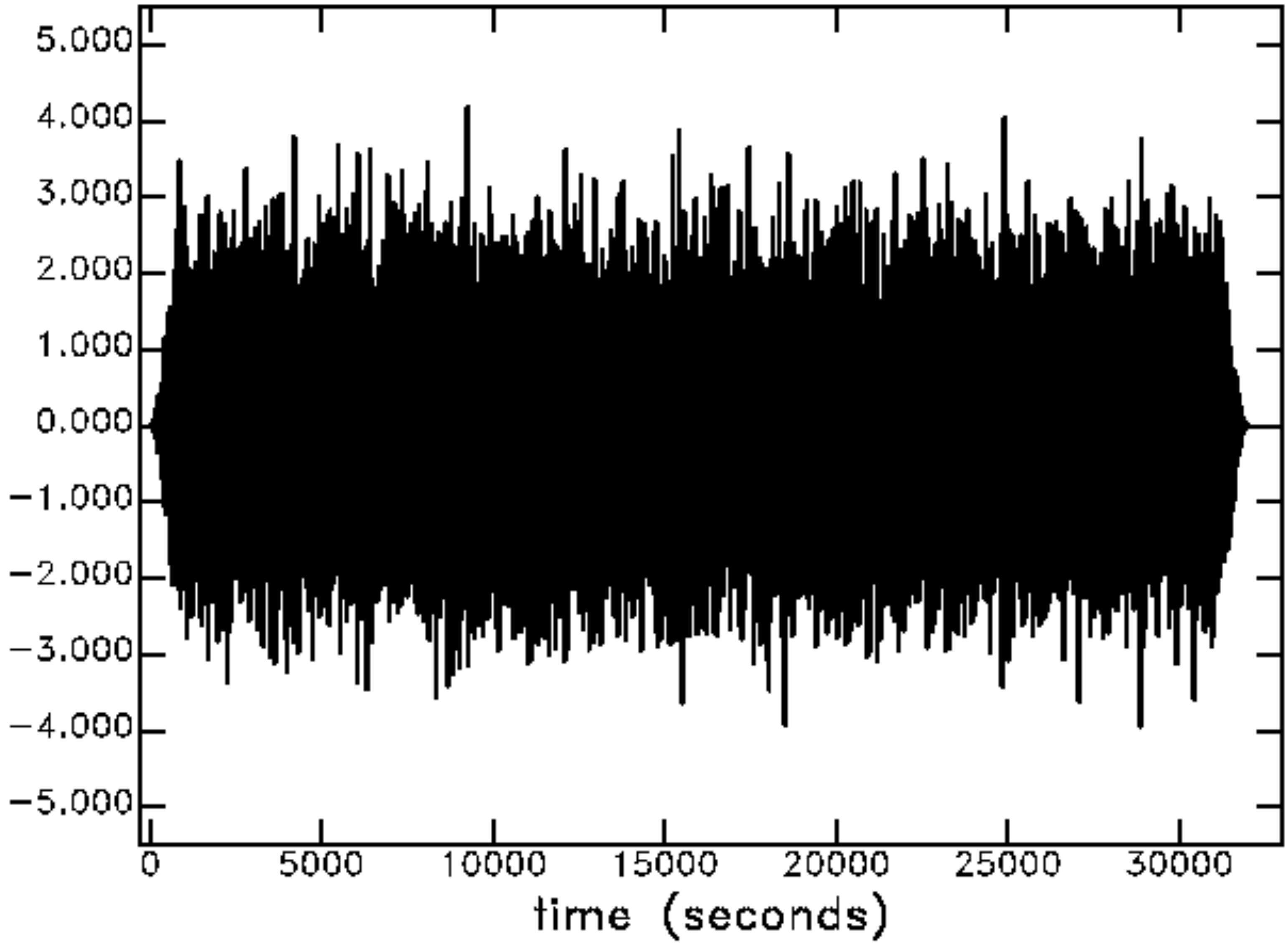
Two stations, P and Q, separated by L km

What is the cross correlation of noise signals recorded at P and Q?

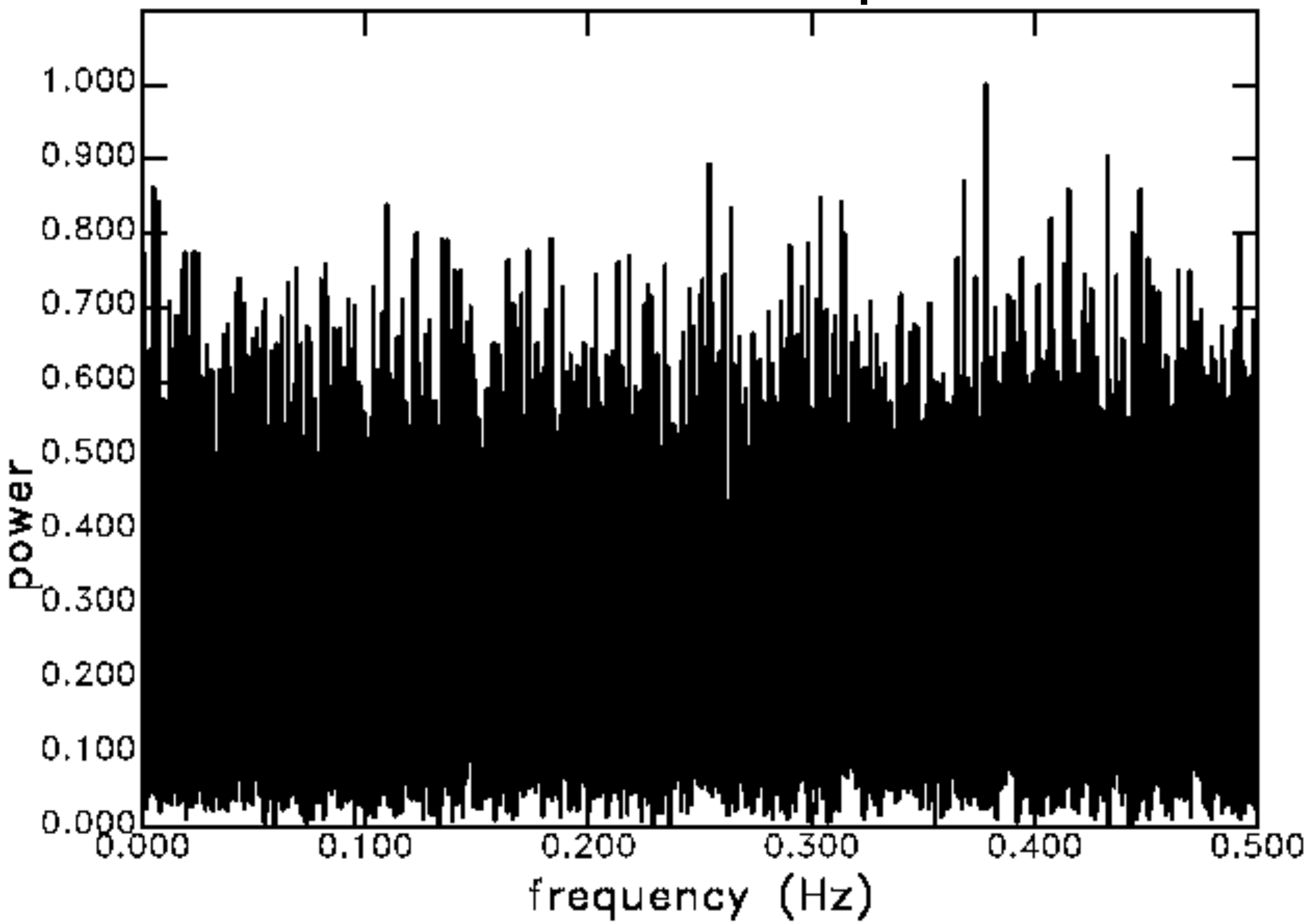


$$R_{PQ}(\tau) = \frac{1}{T}\int_0^T s_P(t)s_Q(t+\tau)dt$$

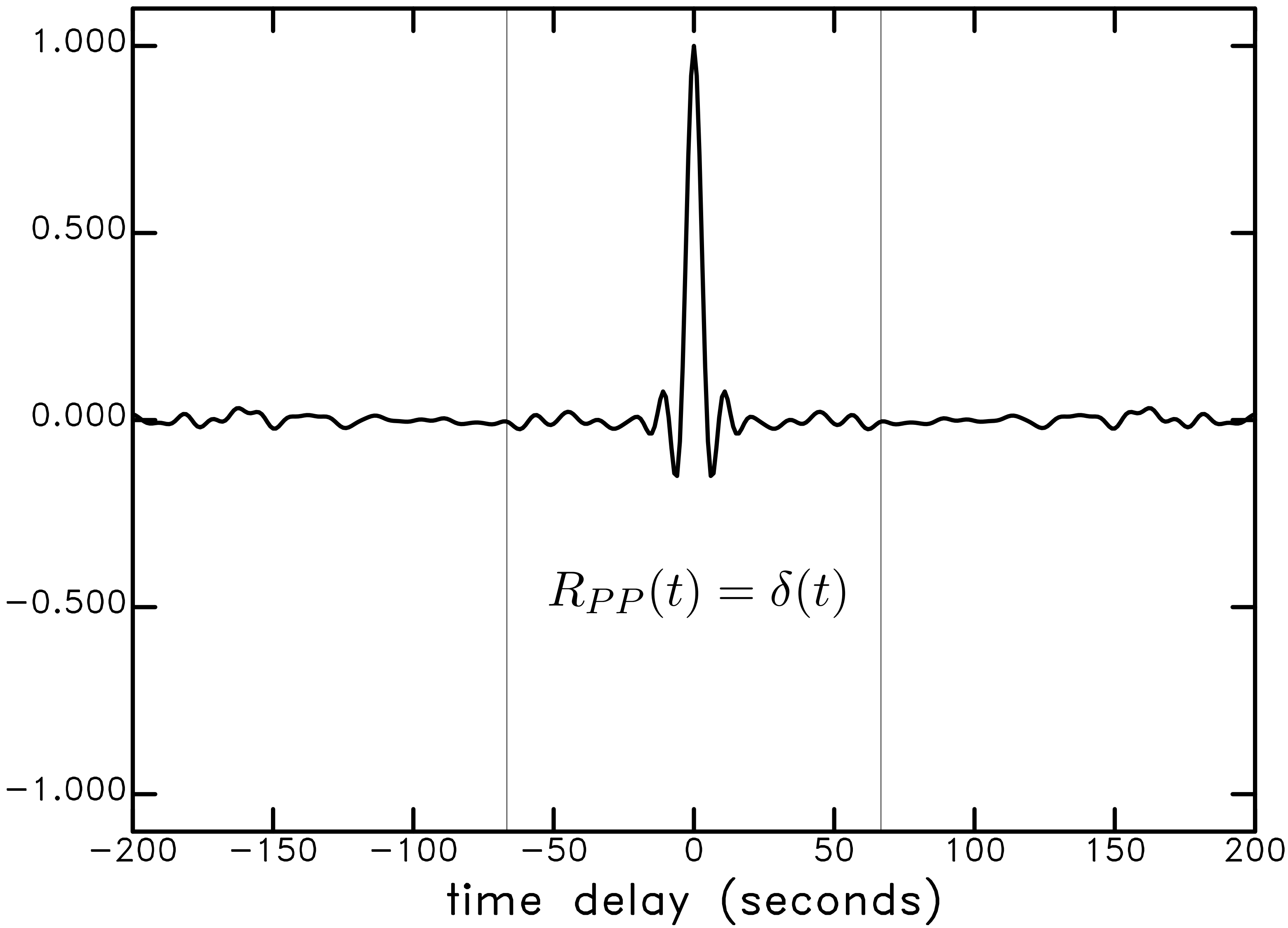
Gaussian white noise, 1 Hz sampling



Gaussian white noise spectrum

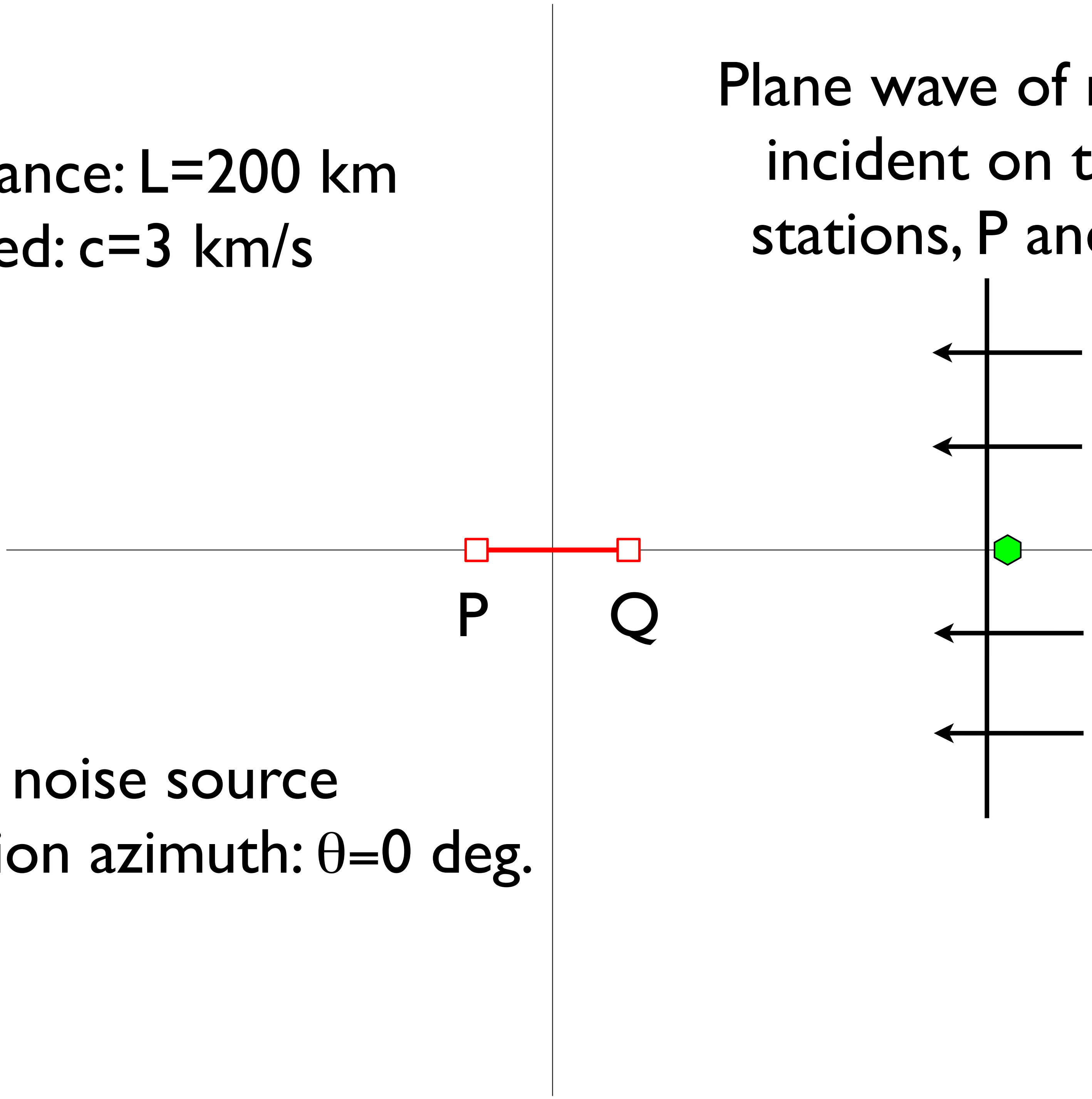


Auto-correlation function of noise



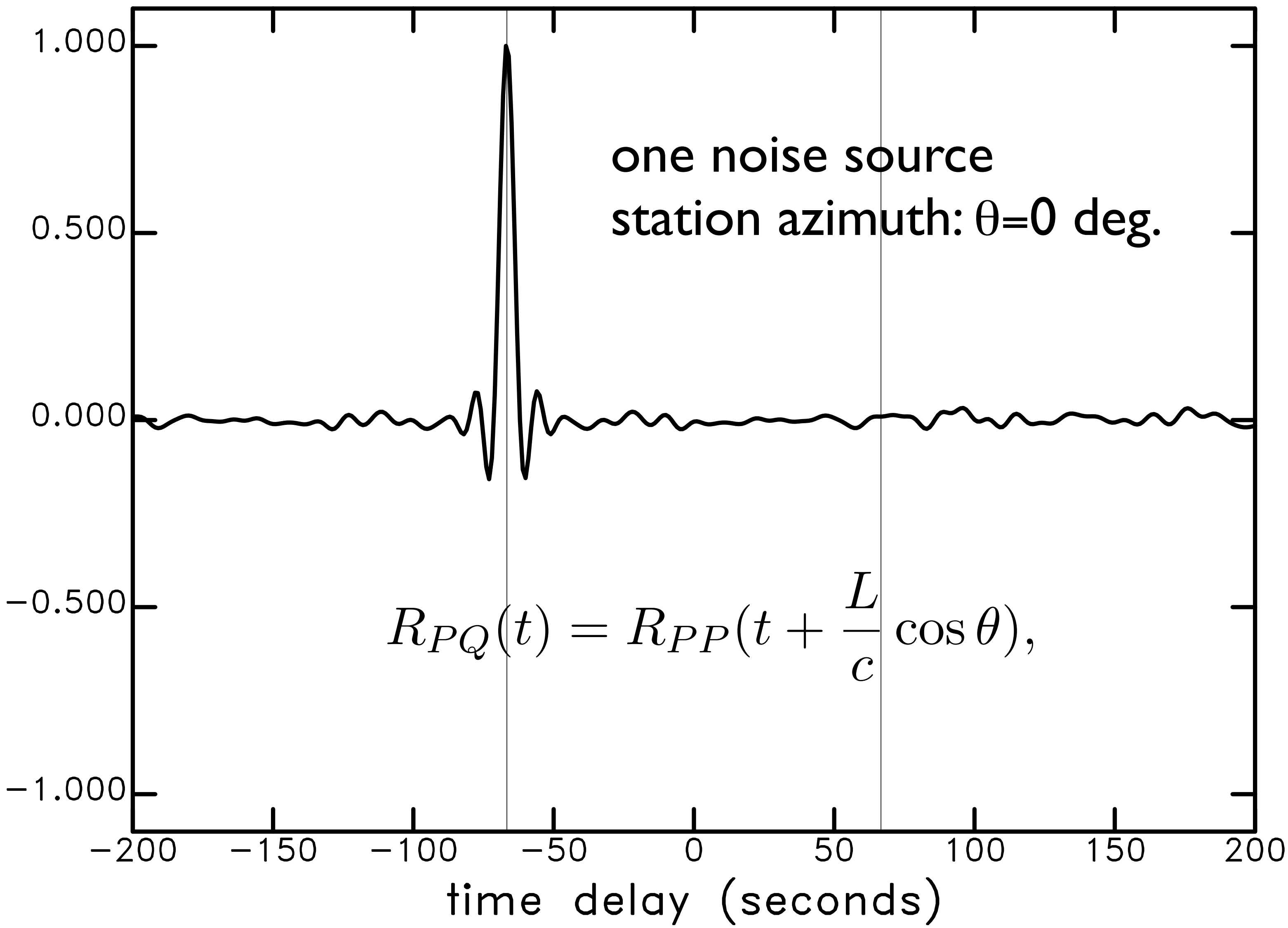
Plane wave of noise incident on two stations, P and Q

distance: L=200 km
speed: c=3 km/s

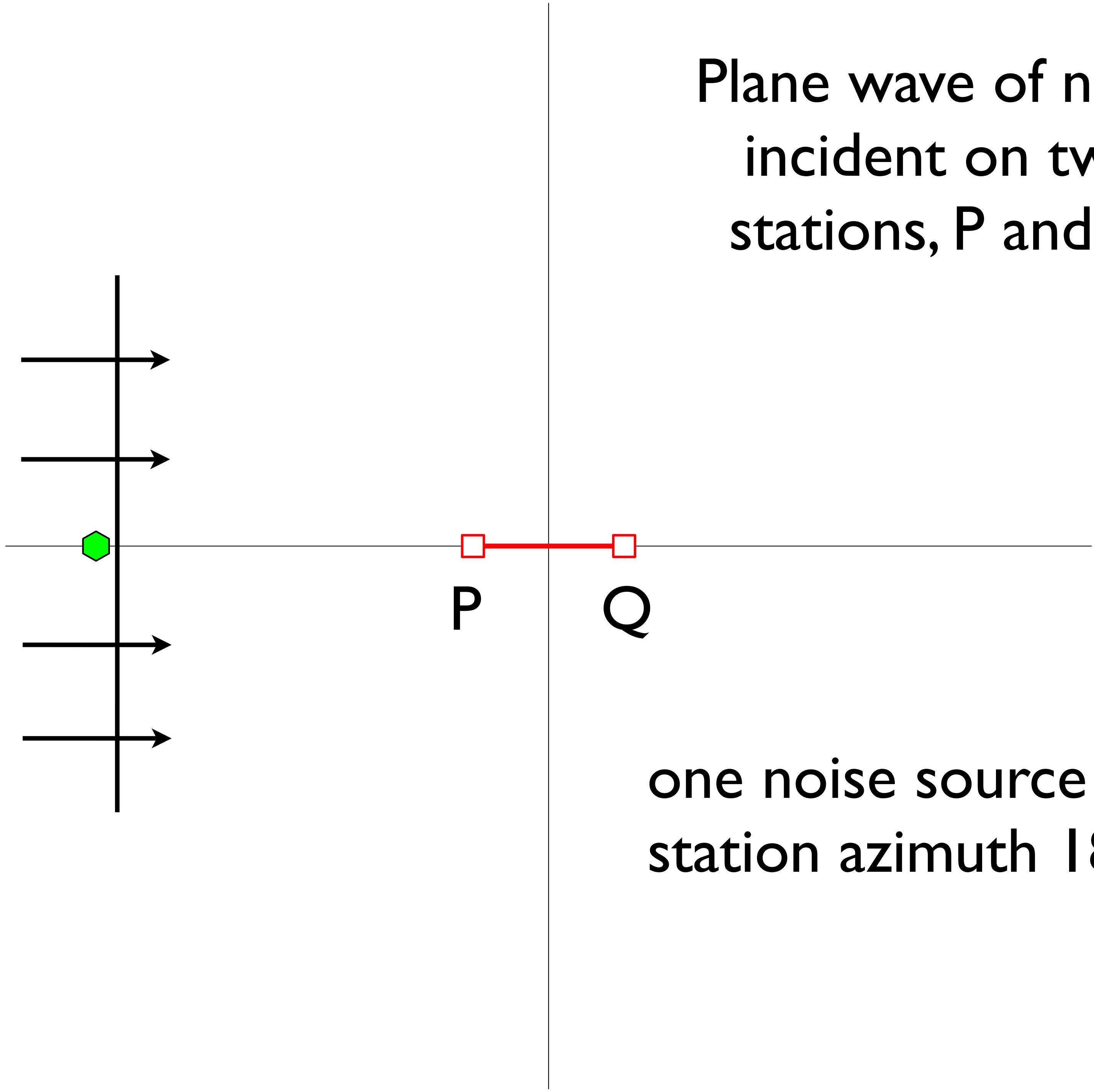


one noise source
station azimuth: θ=0 deg.

Cross-correlation function, P and Q

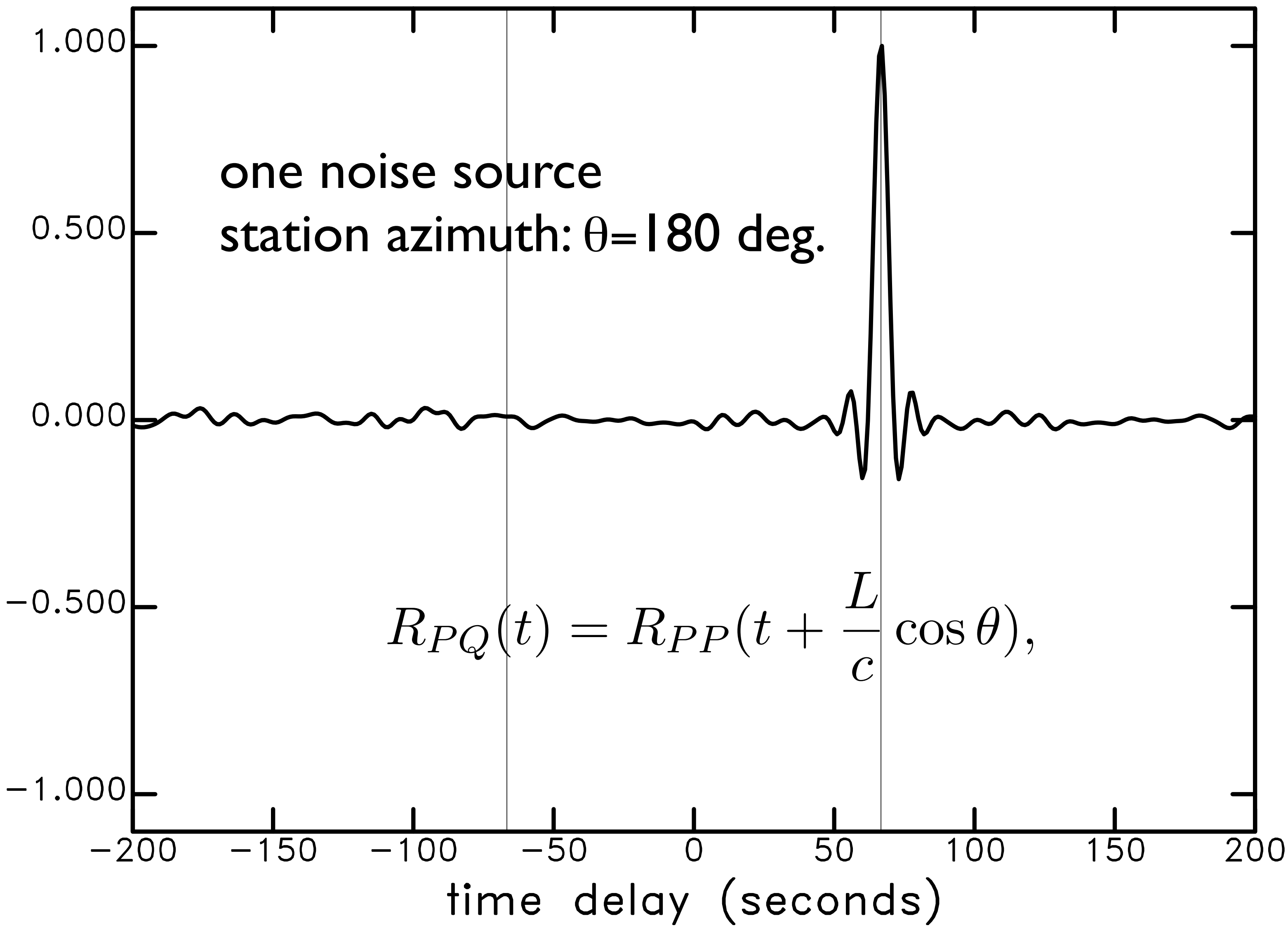


Plane wave of noise incident on two stations, P and Q

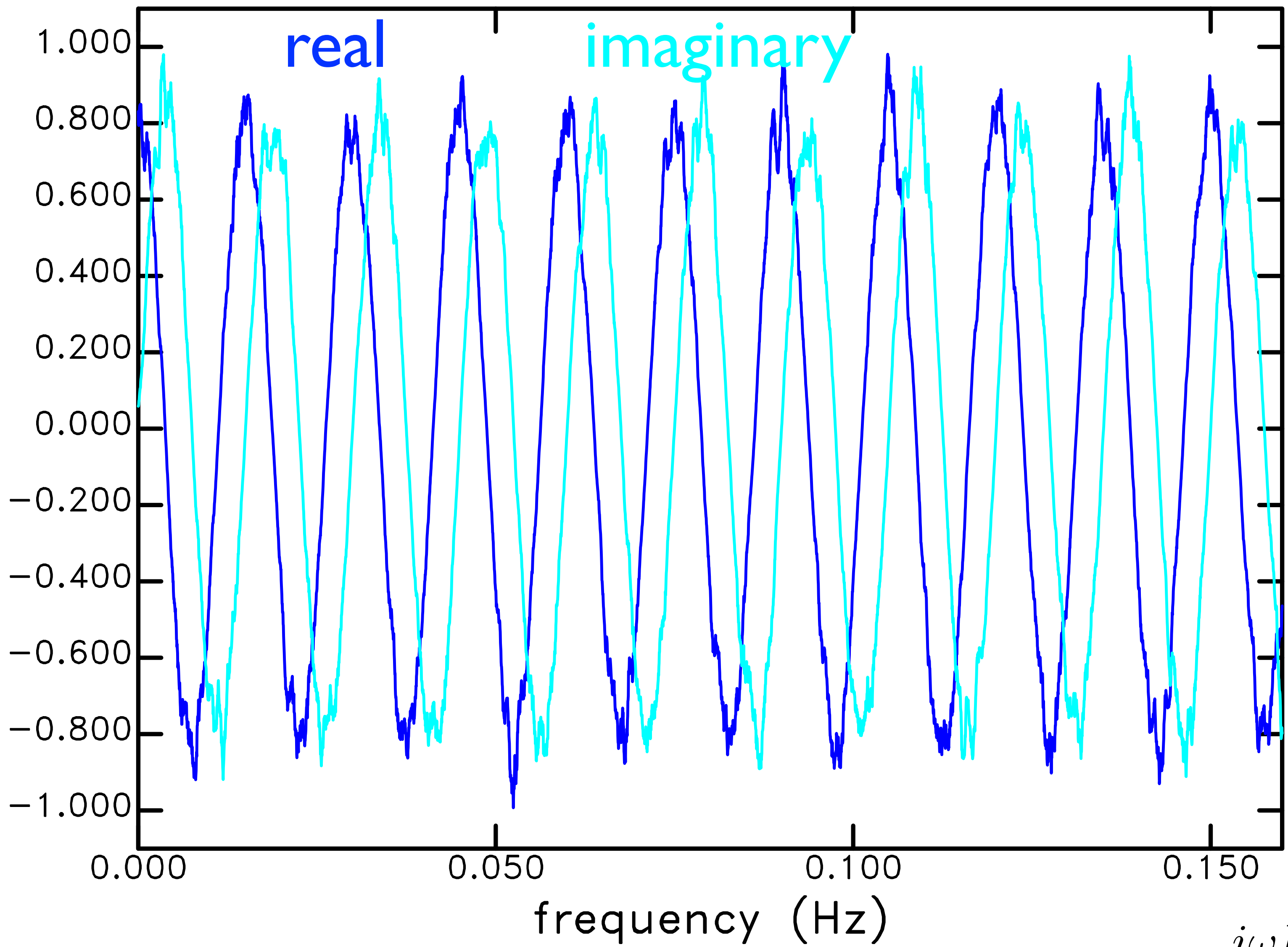


one noise source
station azimuth 180 deg.

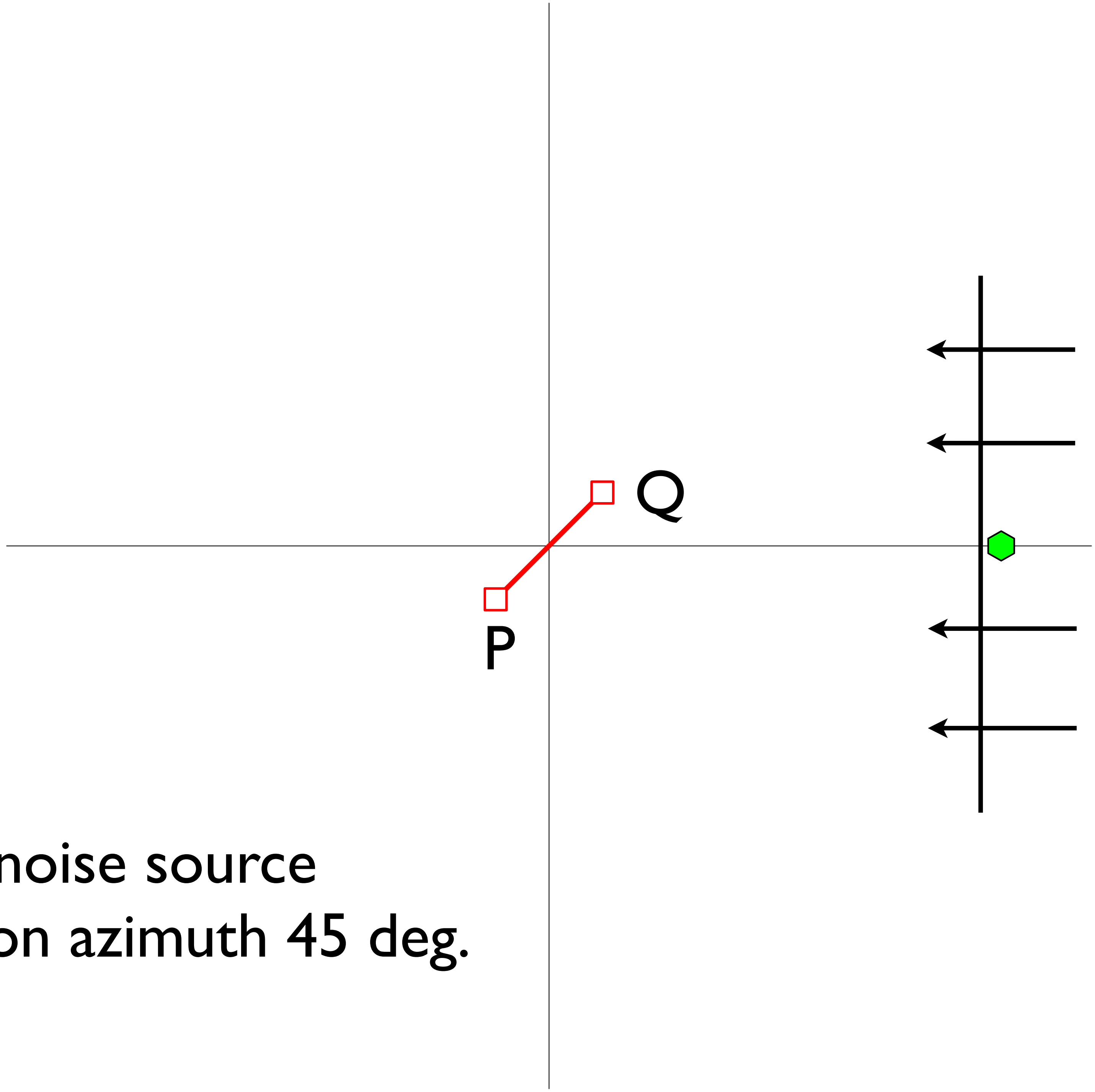
Cross-correlation function, P and Q



Spectrum of cross-correlation function

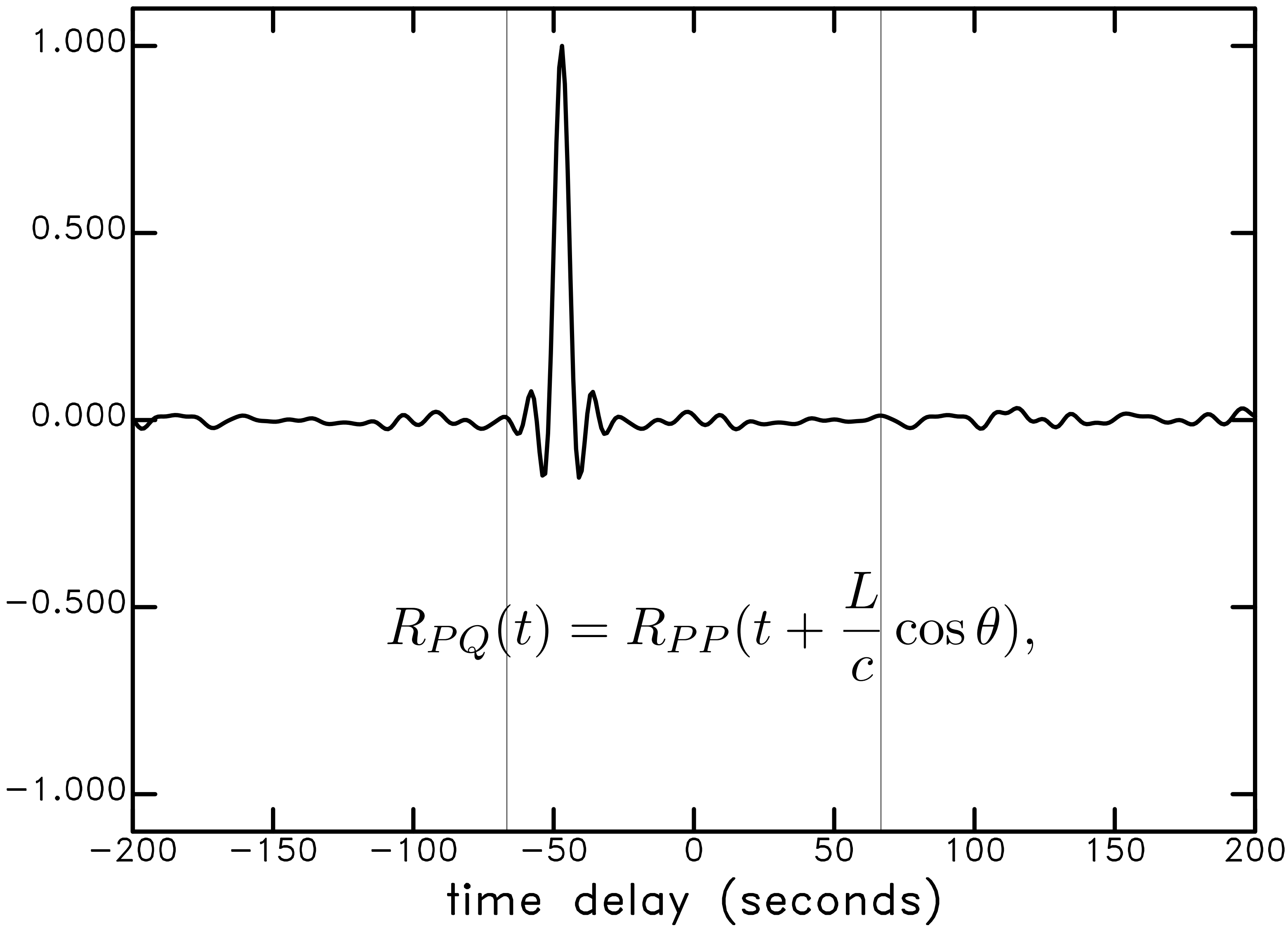


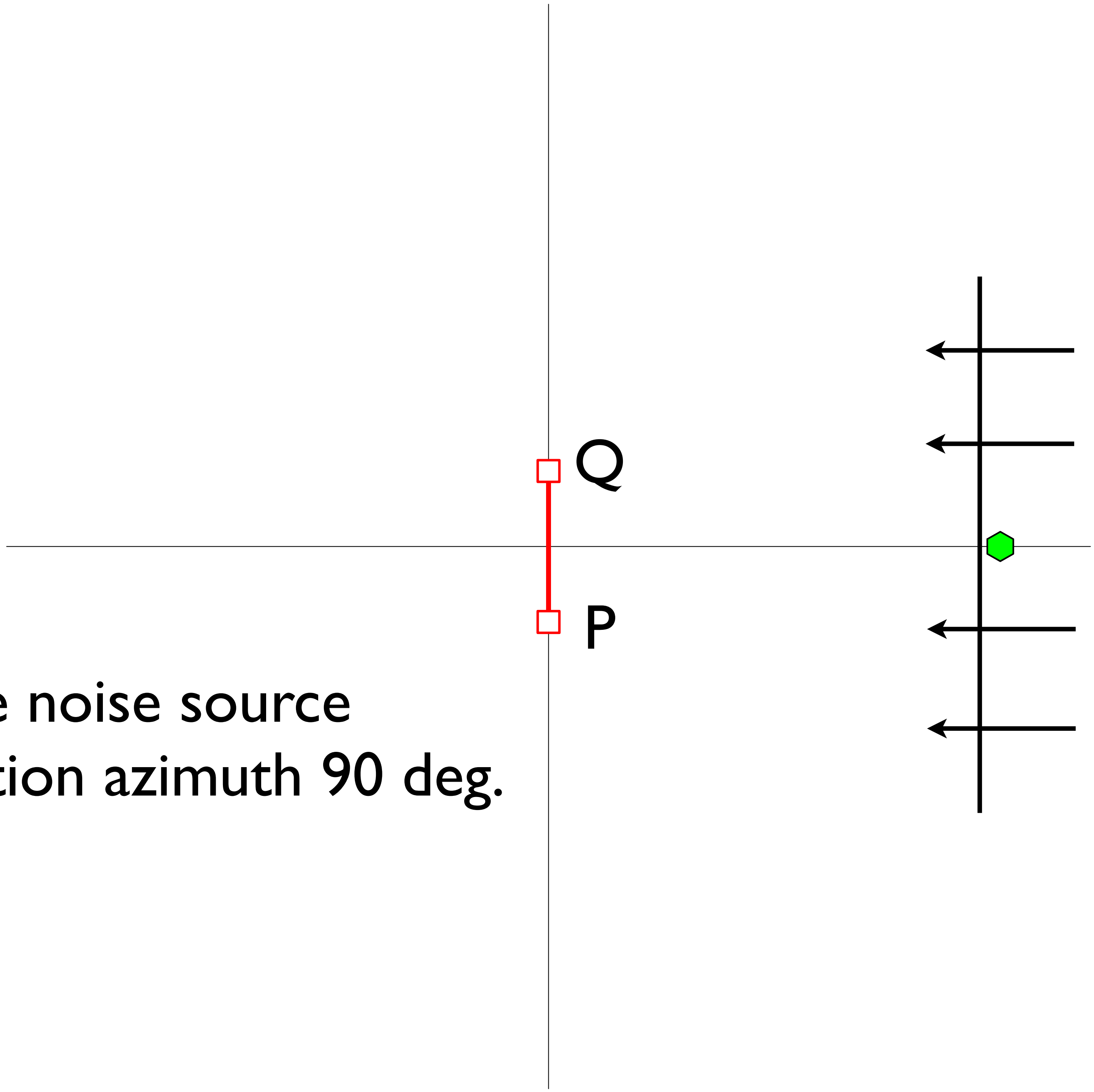
$$s(\omega) = a_0 \exp(\frac{i\omega L \cos(\theta)}{c})$$



one noise source
station azimuth 45 deg.

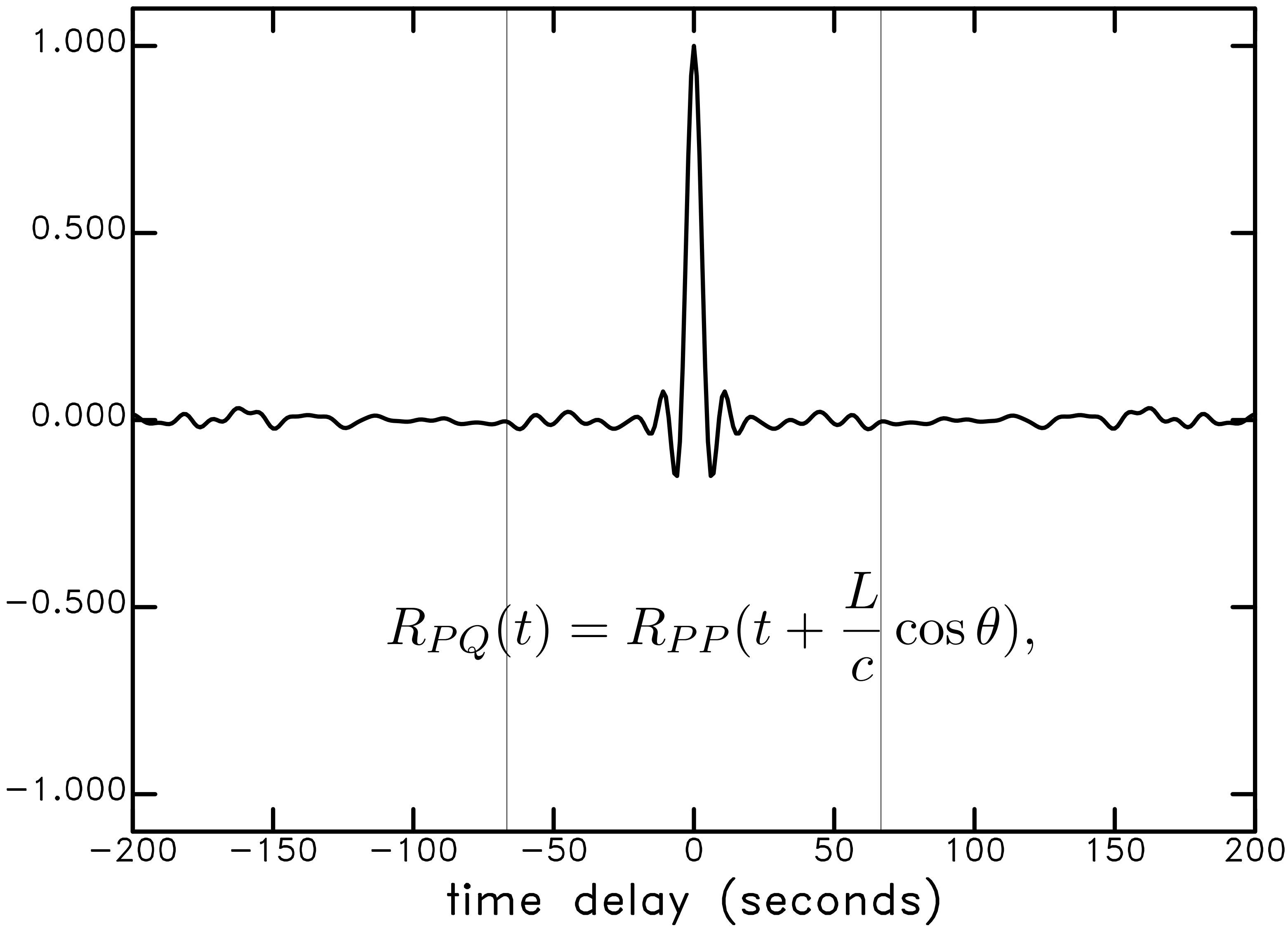
Cross-correlation function, P and Q

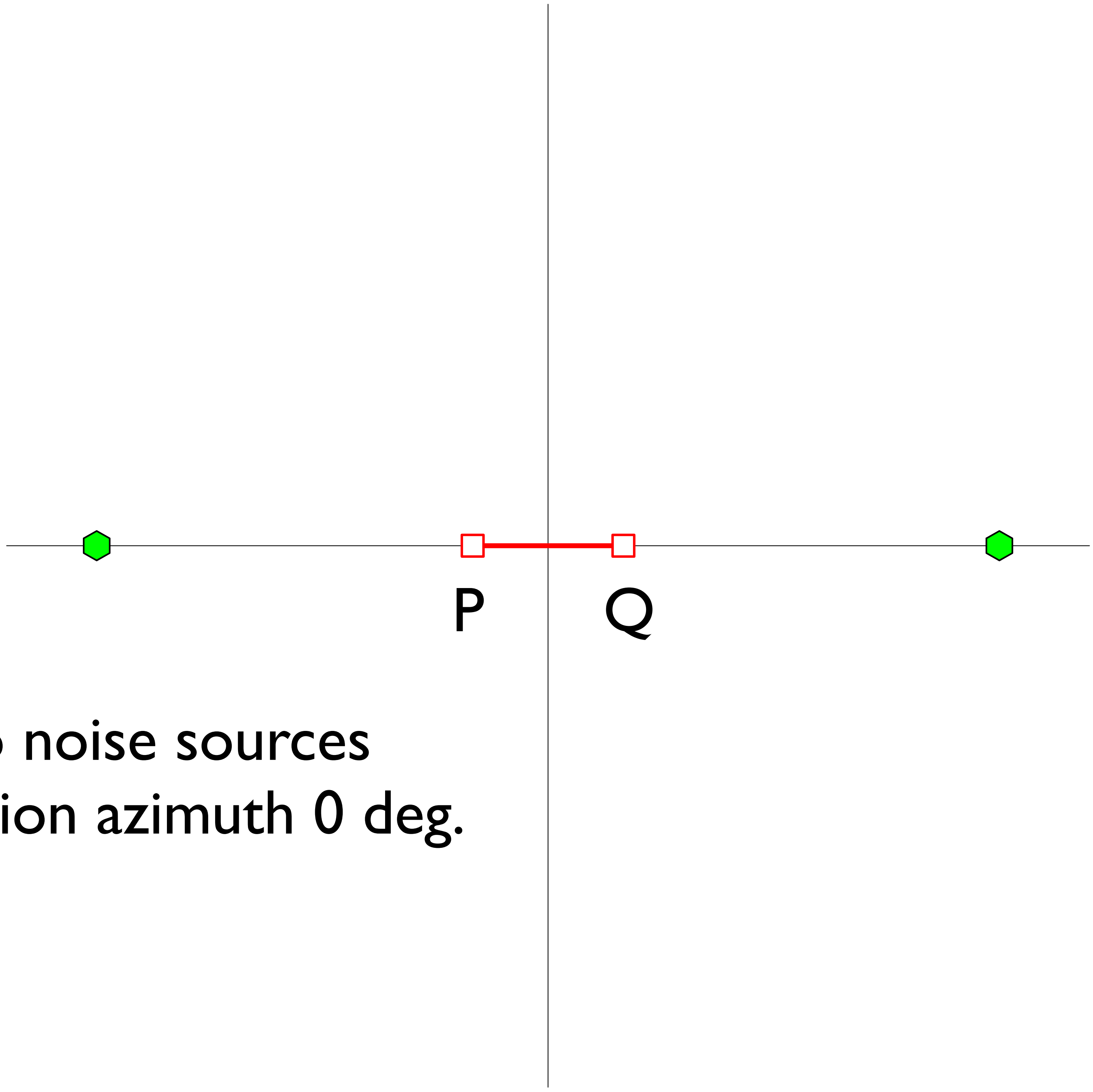




one noise source
station azimuth 90 deg.

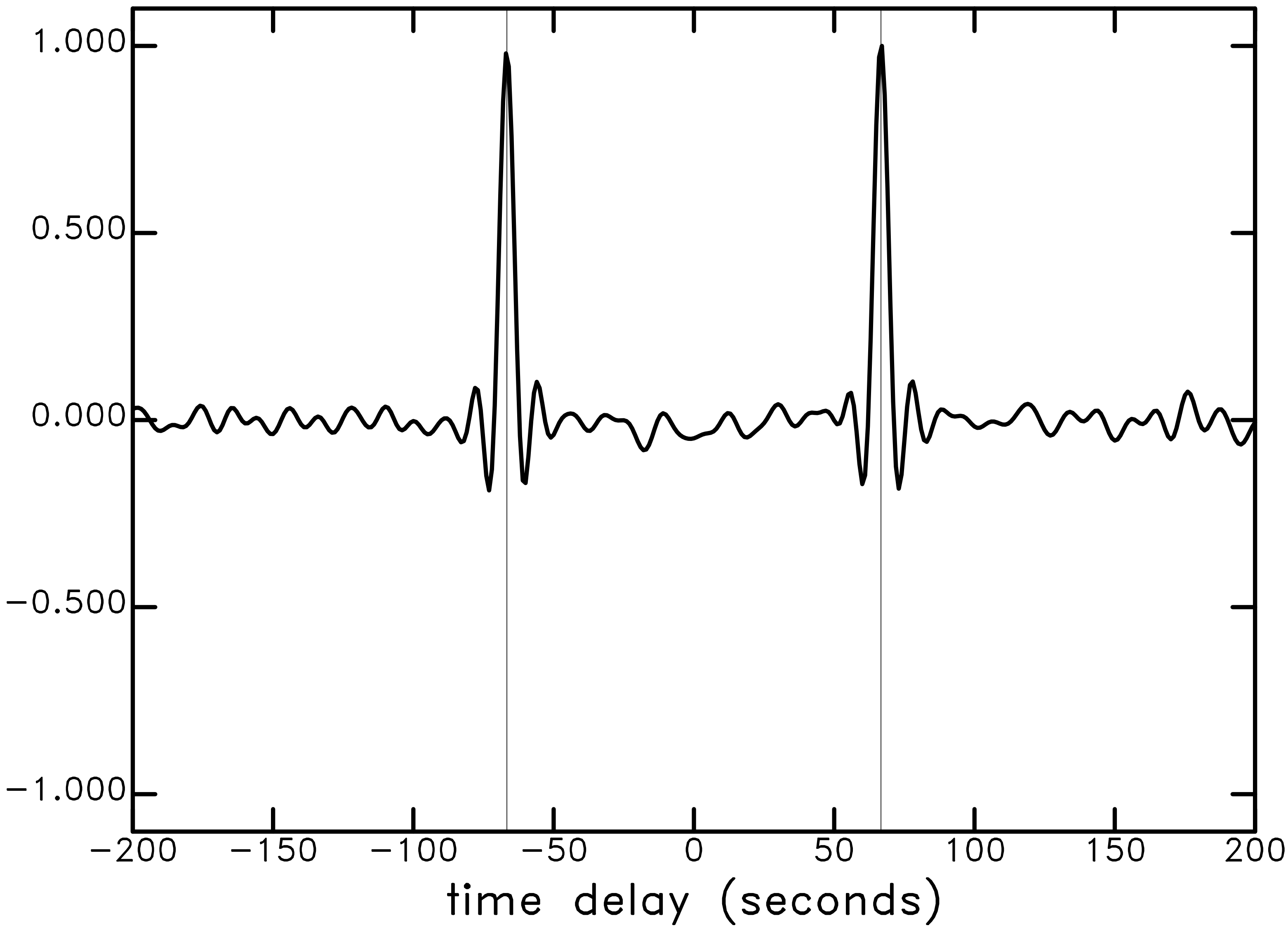
Cross-correlation function, P and Q

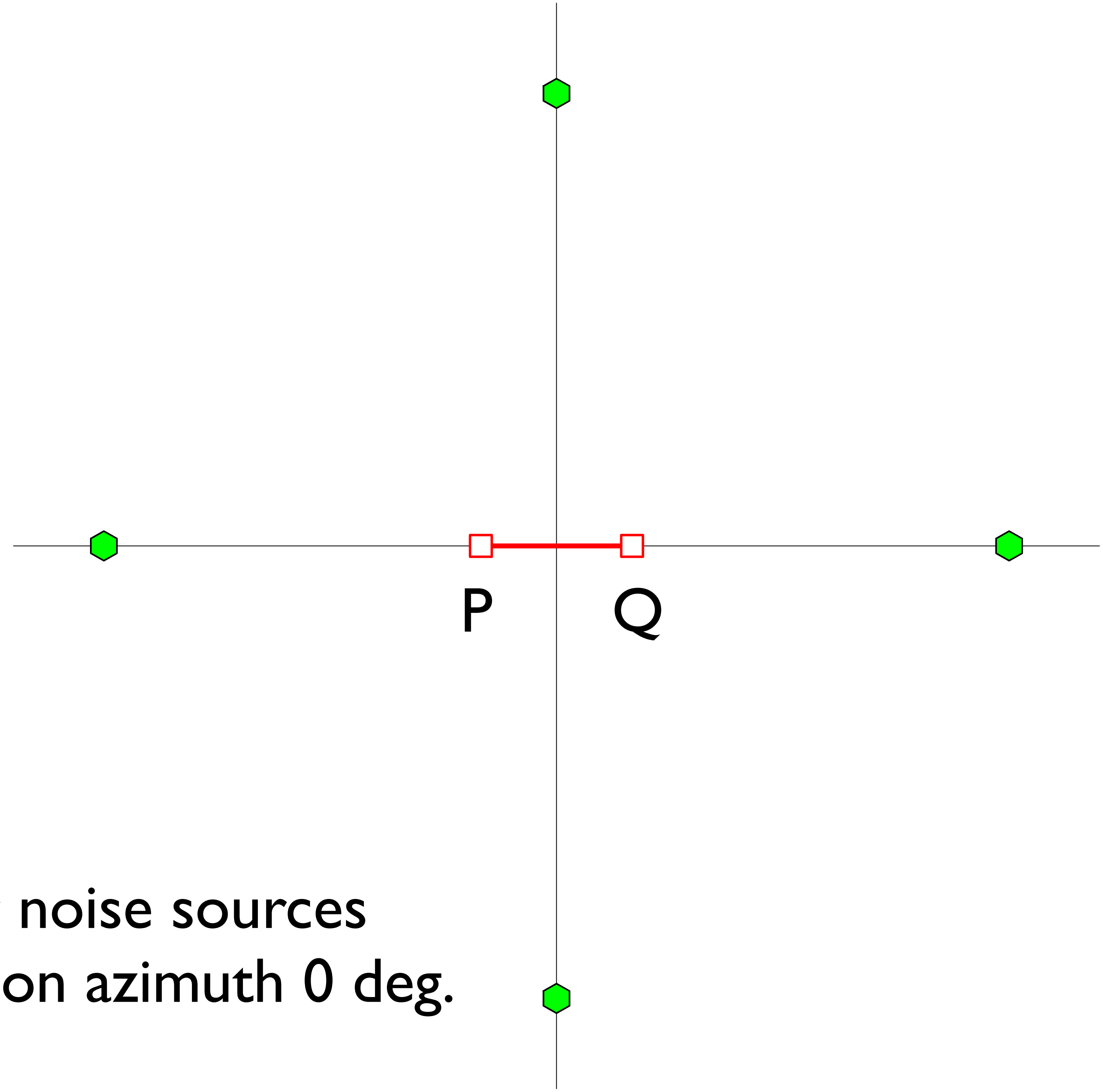




two noise sources
station azimuth 0 deg.

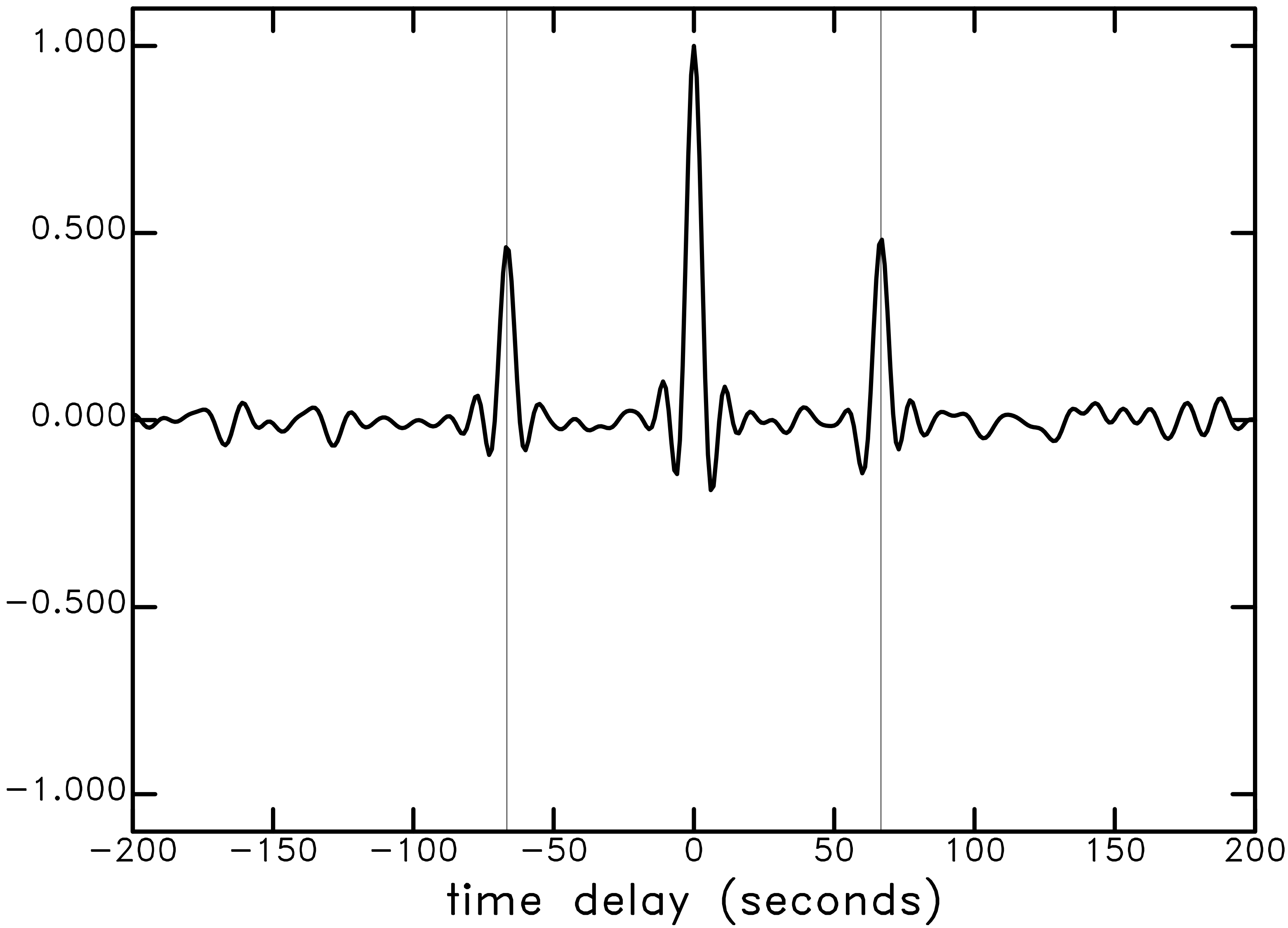
Cross-correlation function, P and Q



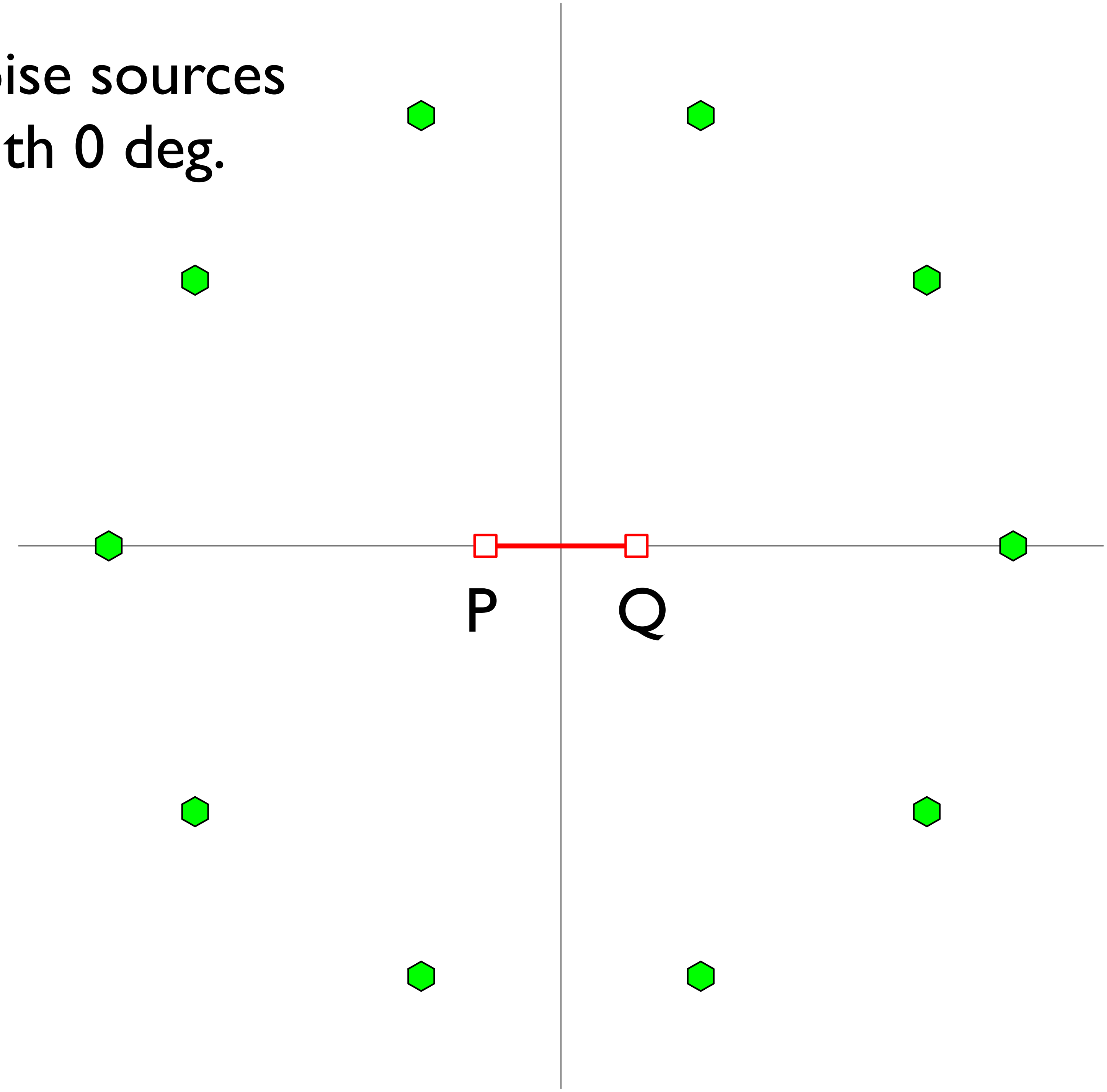


four noise sources
station azimuth 0 deg.

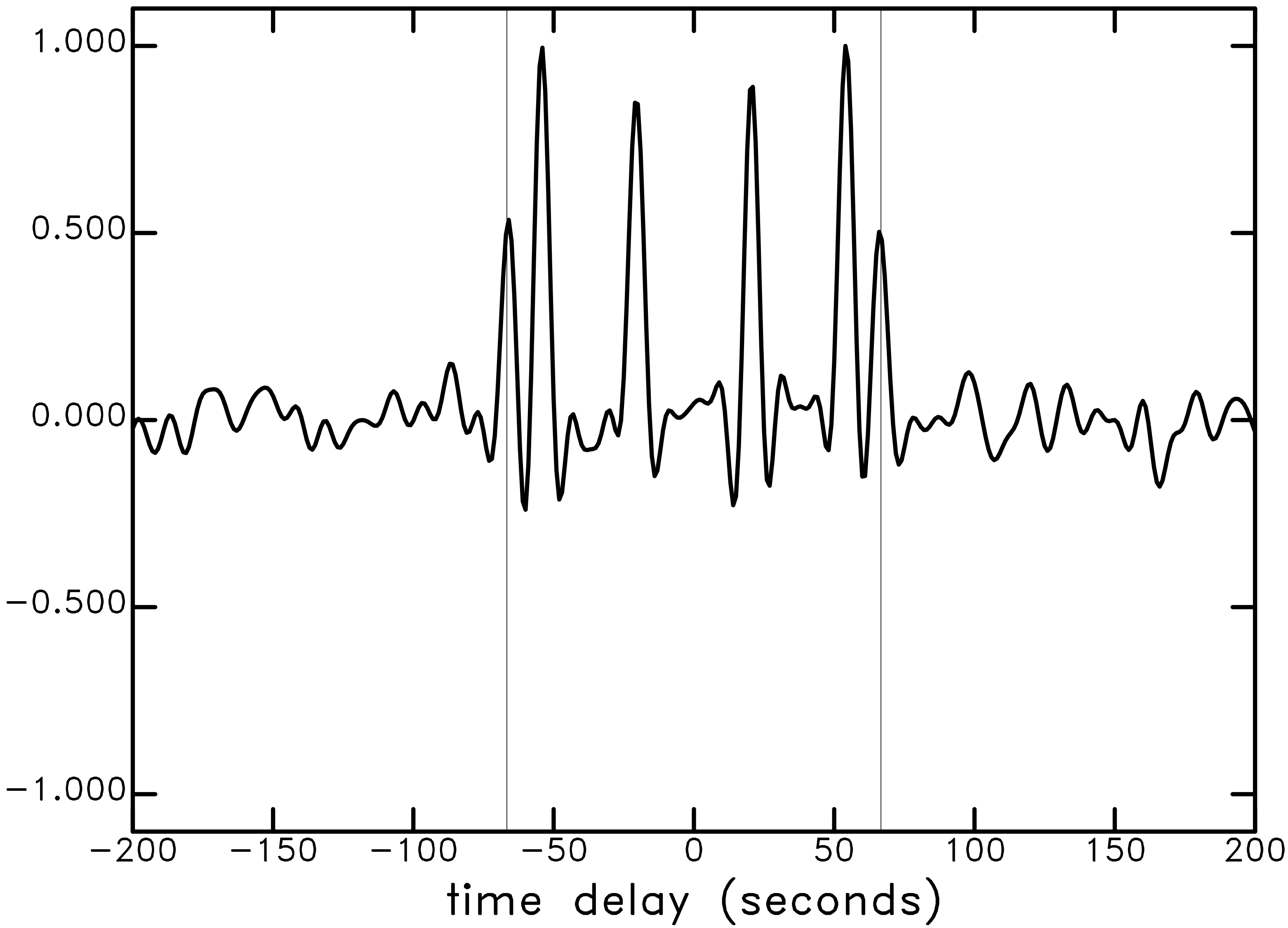
Cross-correlation function, P and Q



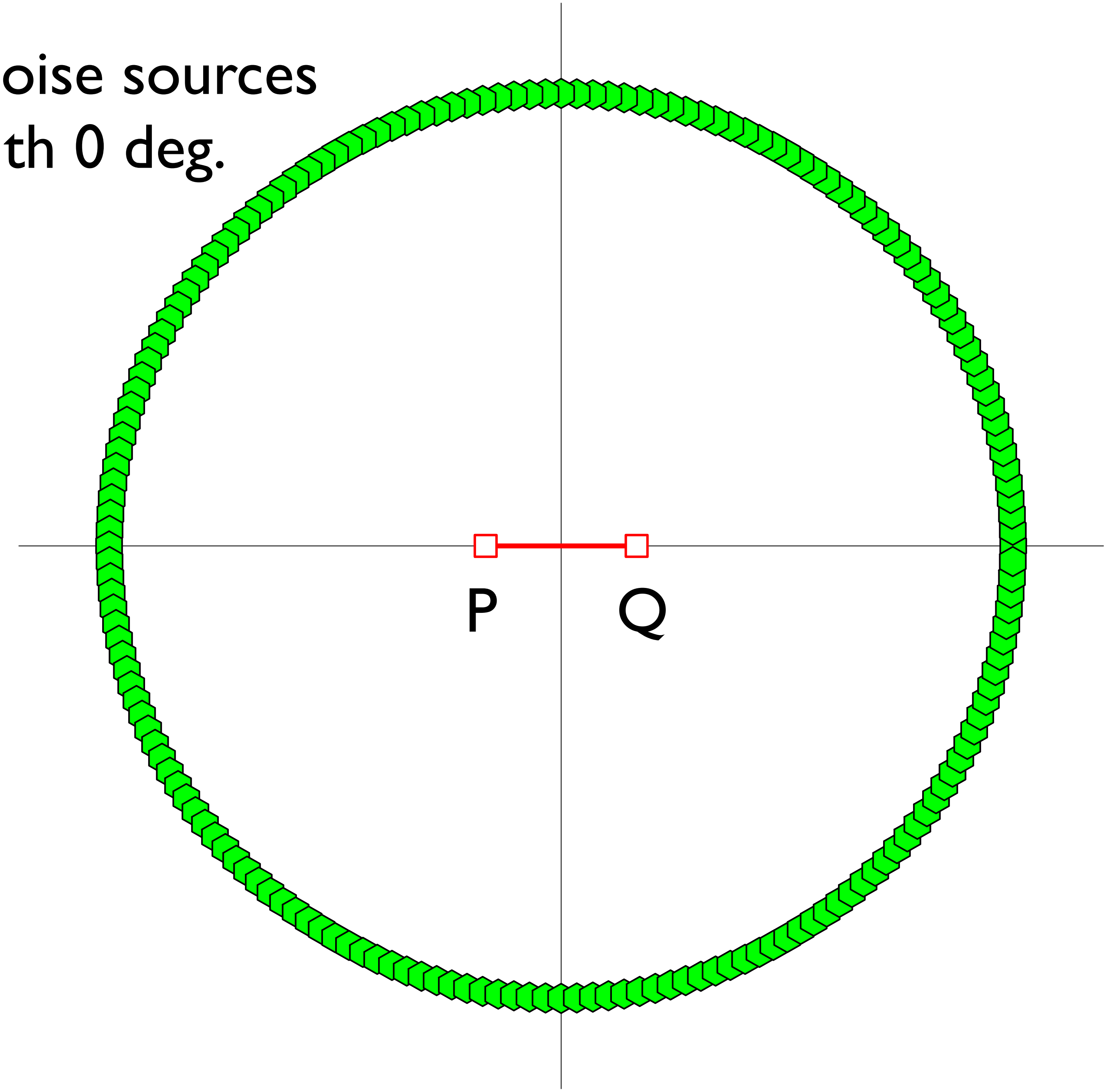
10 noise sources
azimuth 0 deg.



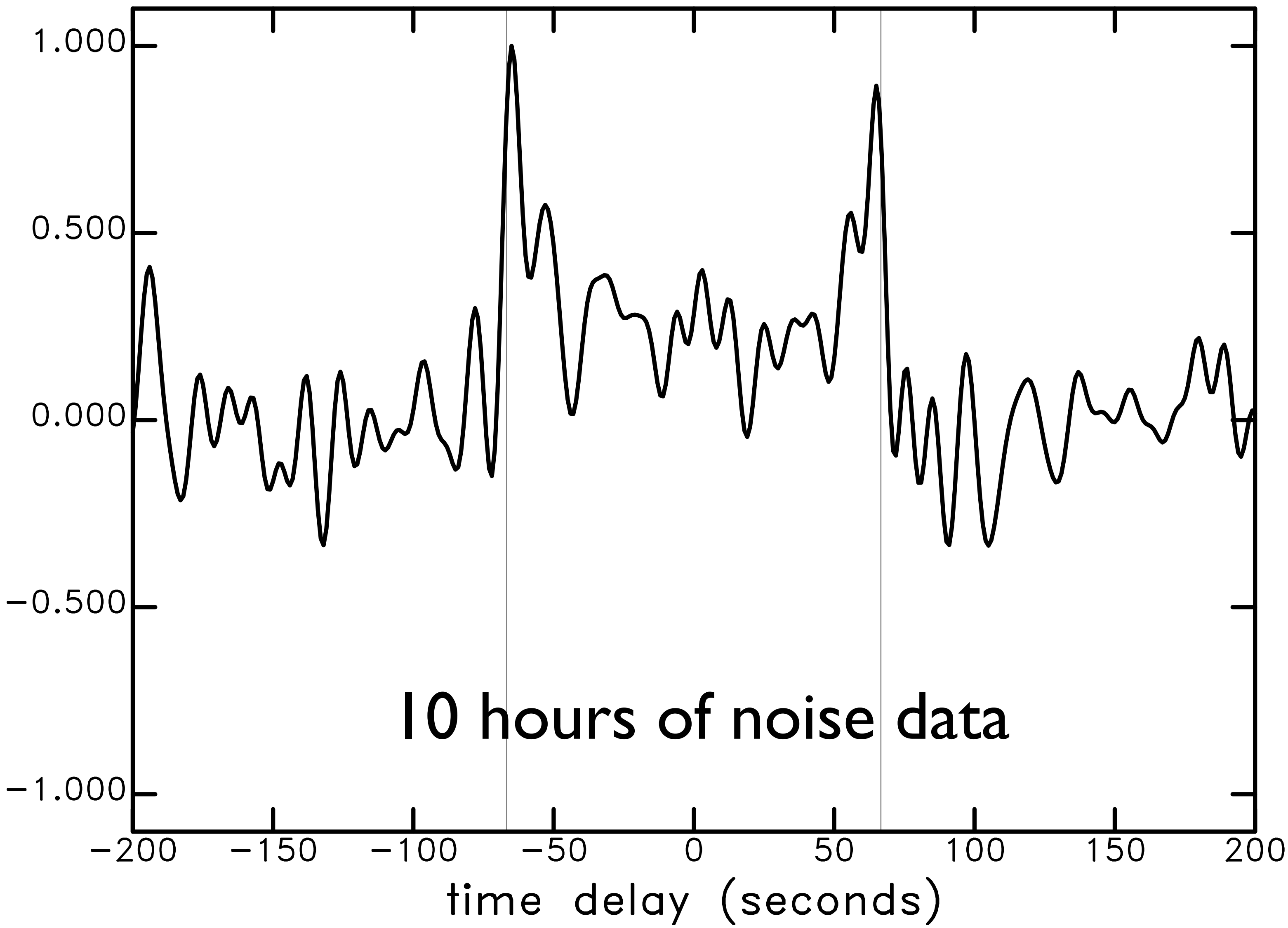
Cross-correlation function, P and Q



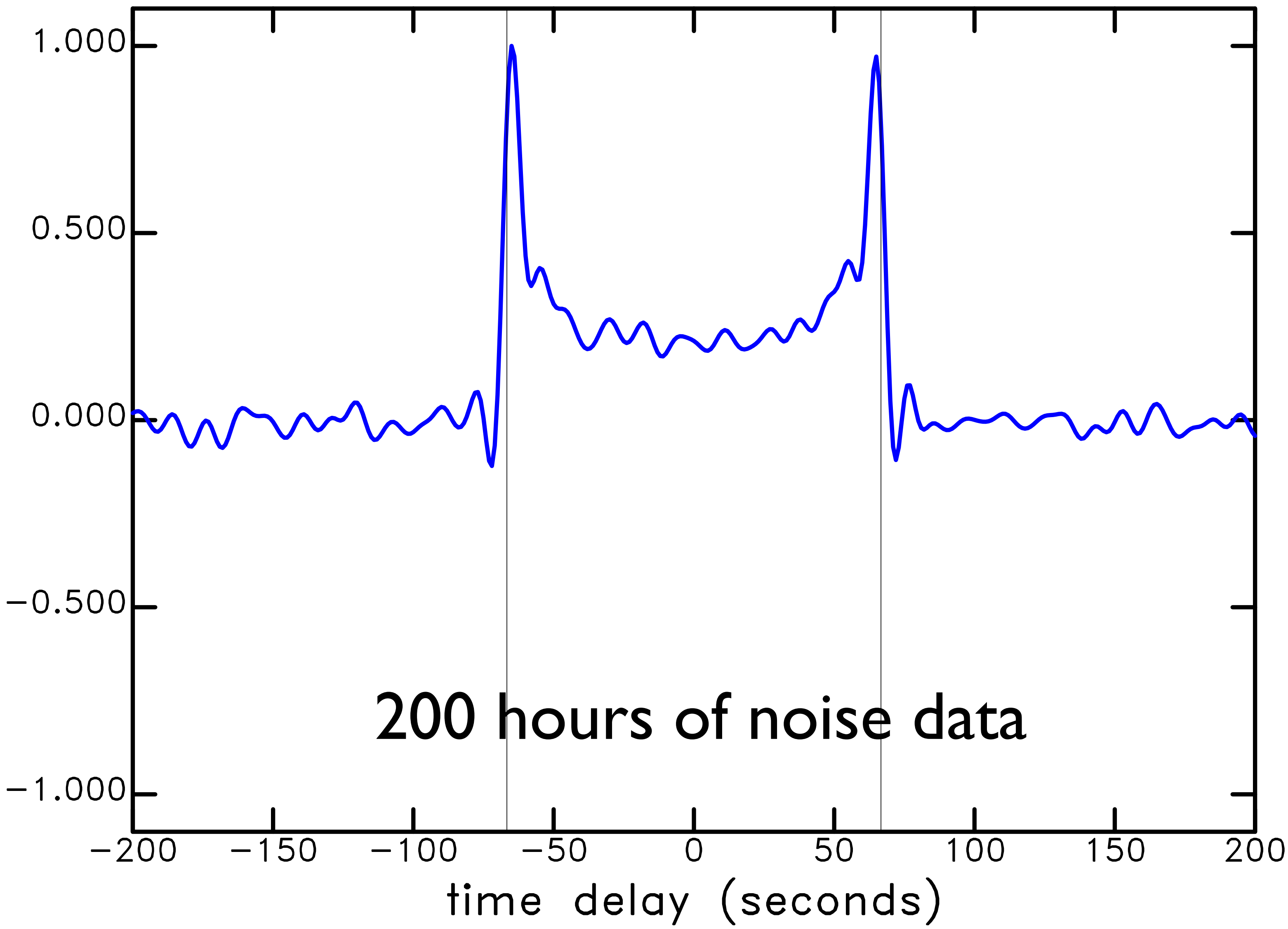
180 noise sources
azimuth 0 deg.

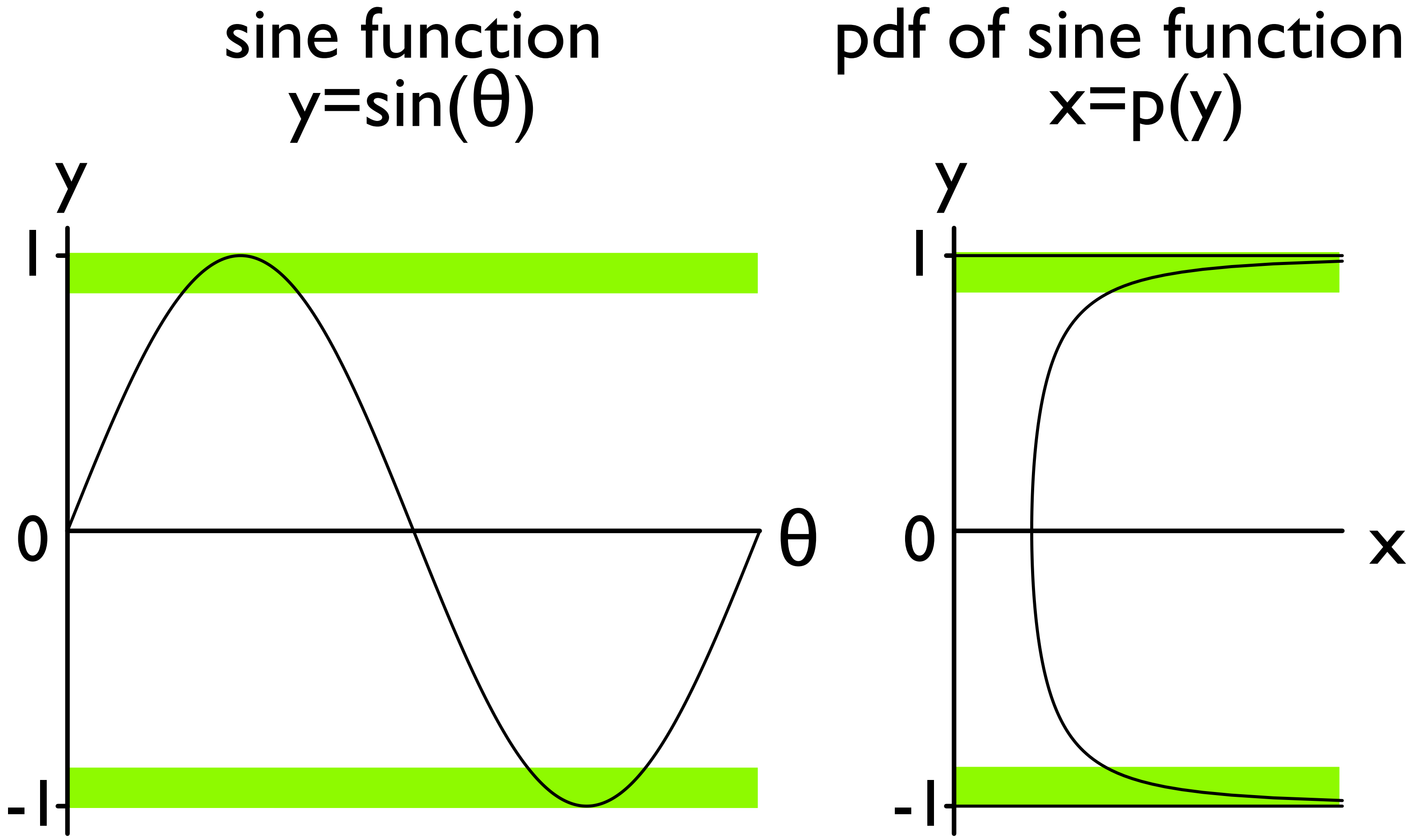


Cross-correlation function, P and Q

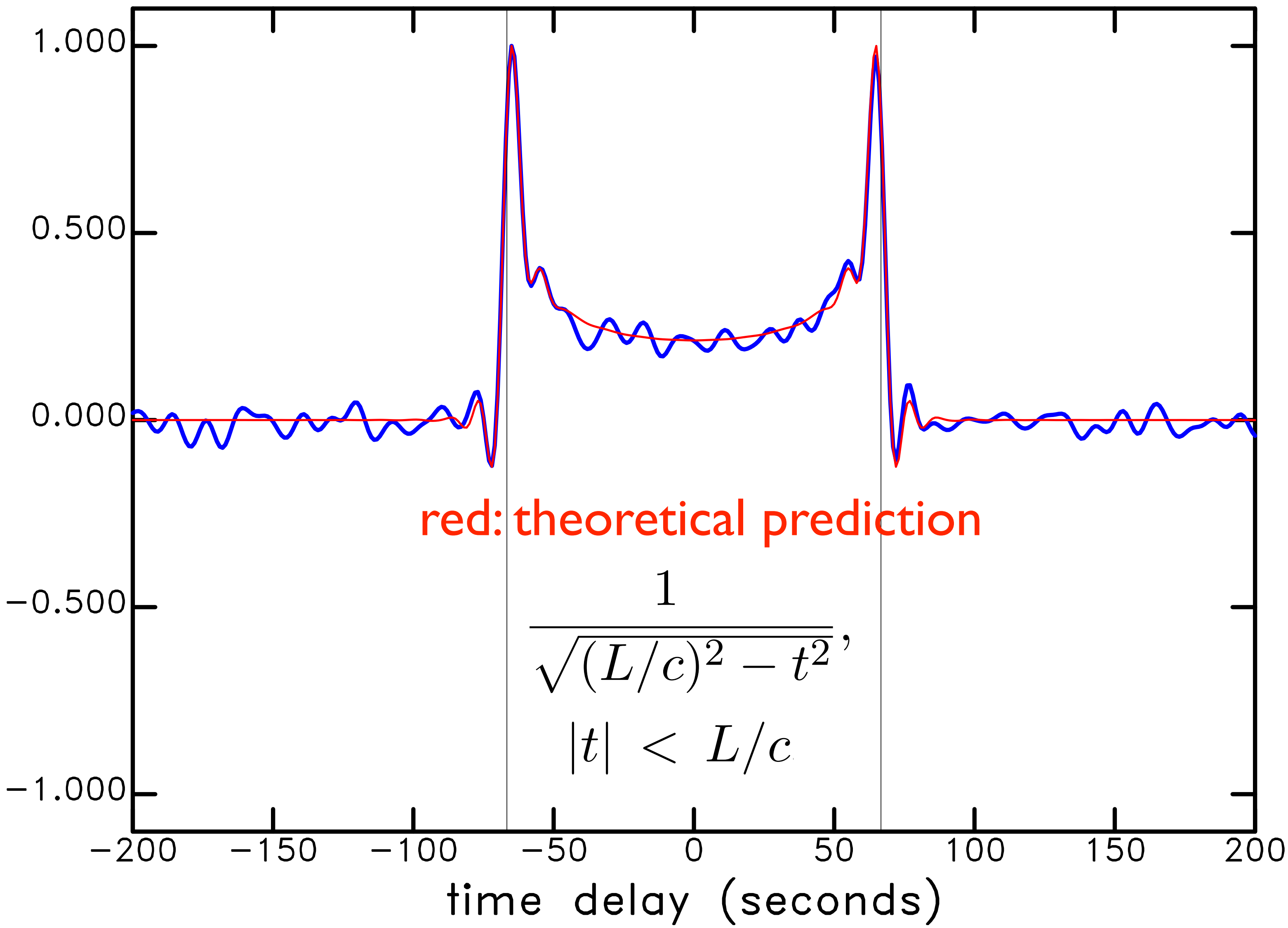


Cross-correlation function, P and Q




sine function
y=sin(θ)
pdf of sine function
x=p(y)
y
1
0
-1
θ
y
1
0
-1
x

Cross-correlation function, P and Q

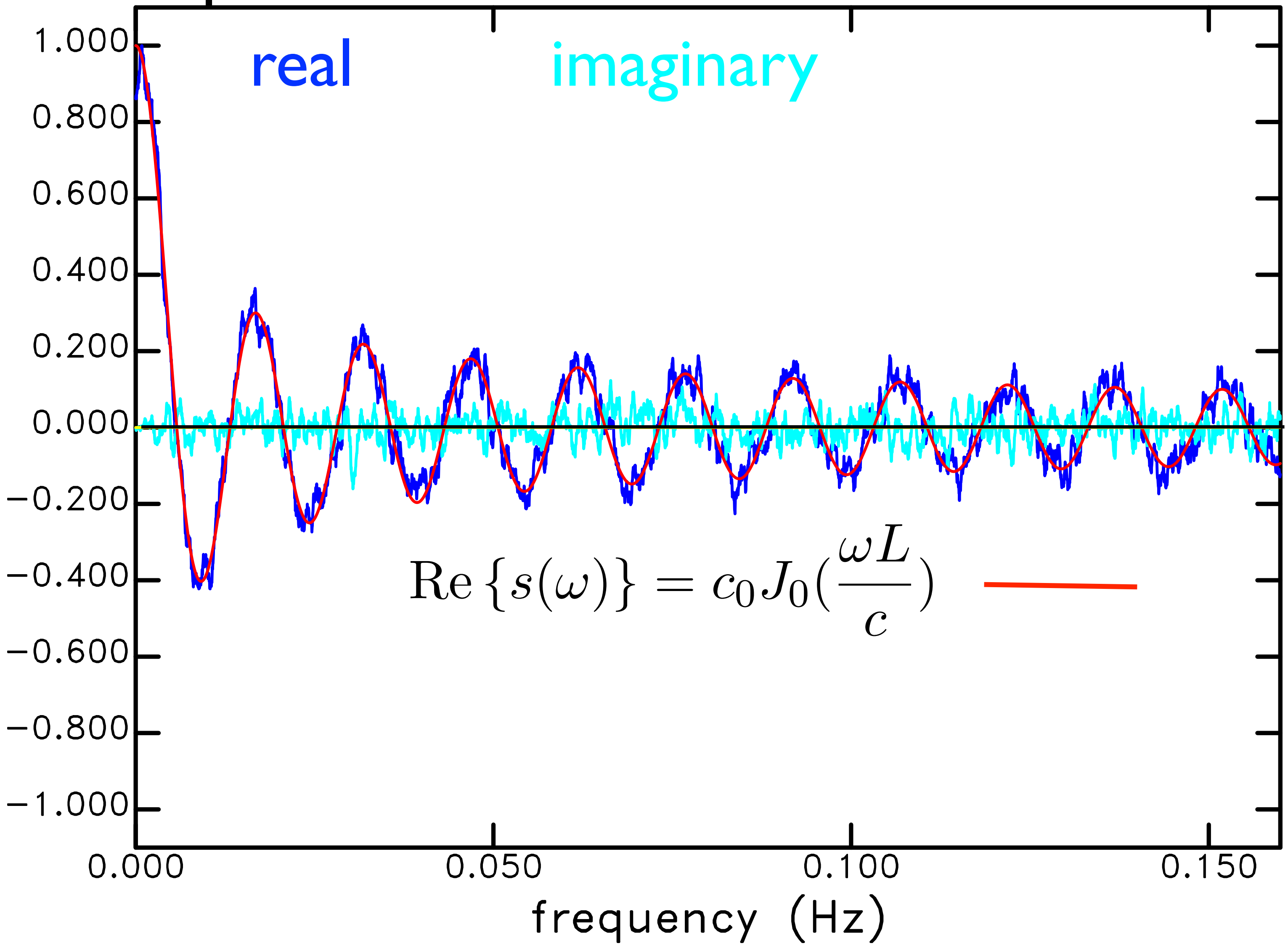


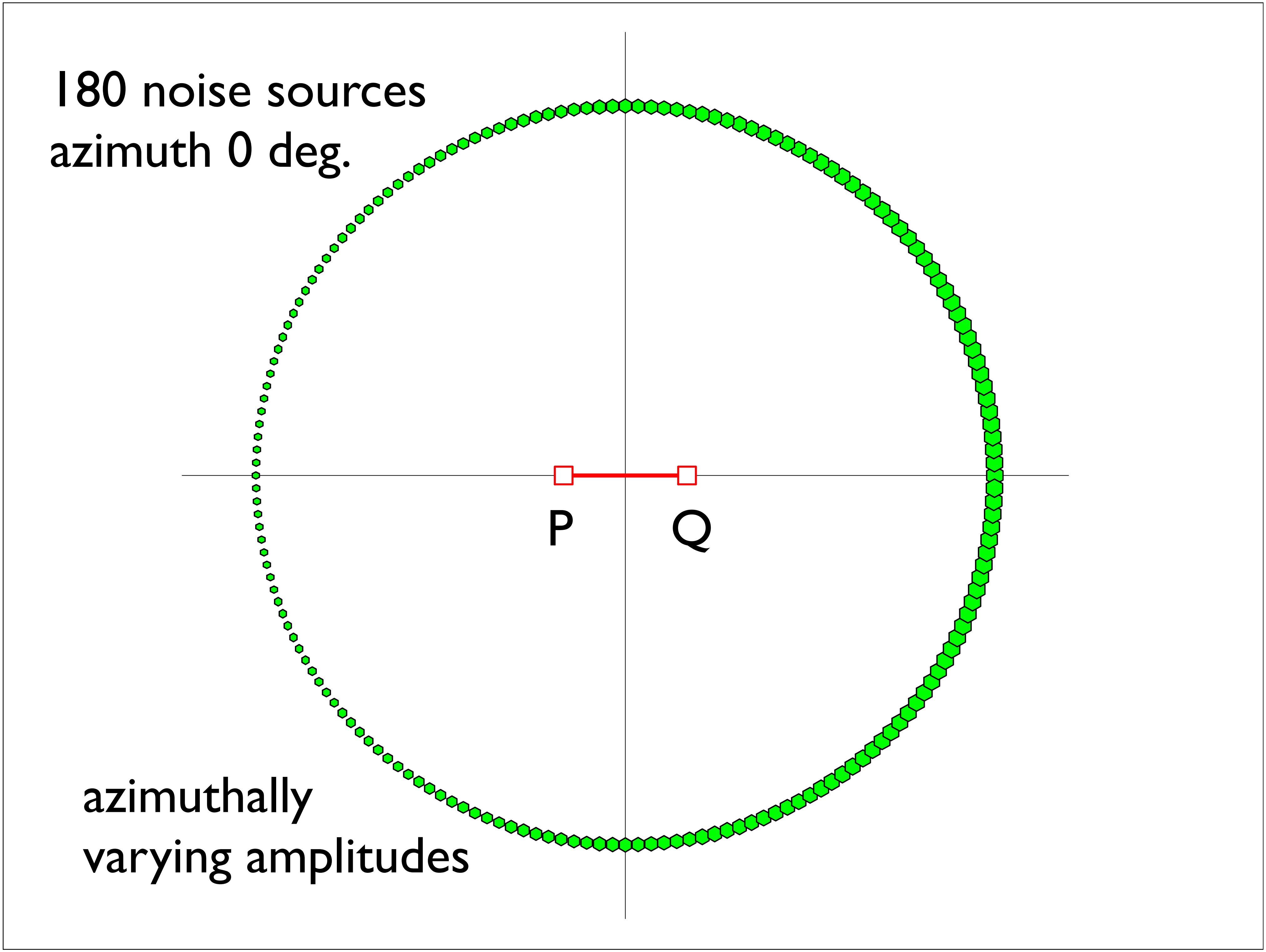
What about the Fourier transform?

What about the Fourier transform?

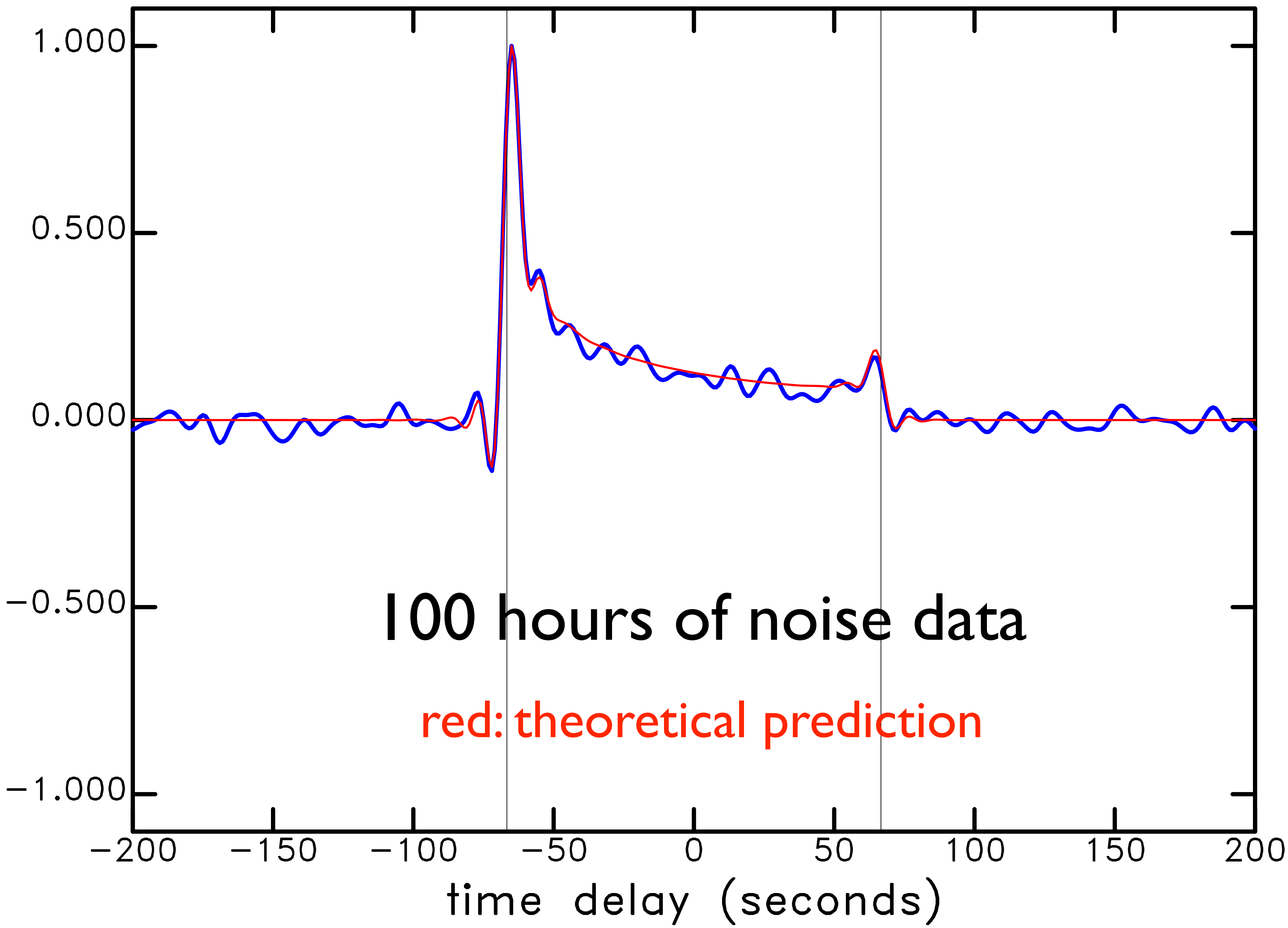
$$\frac{1}{\sqrt{(L/c)^2 - t^2}} \longrightarrow J_0(\frac{\omega L}{c})$$

Spectrum of cross-correlation function

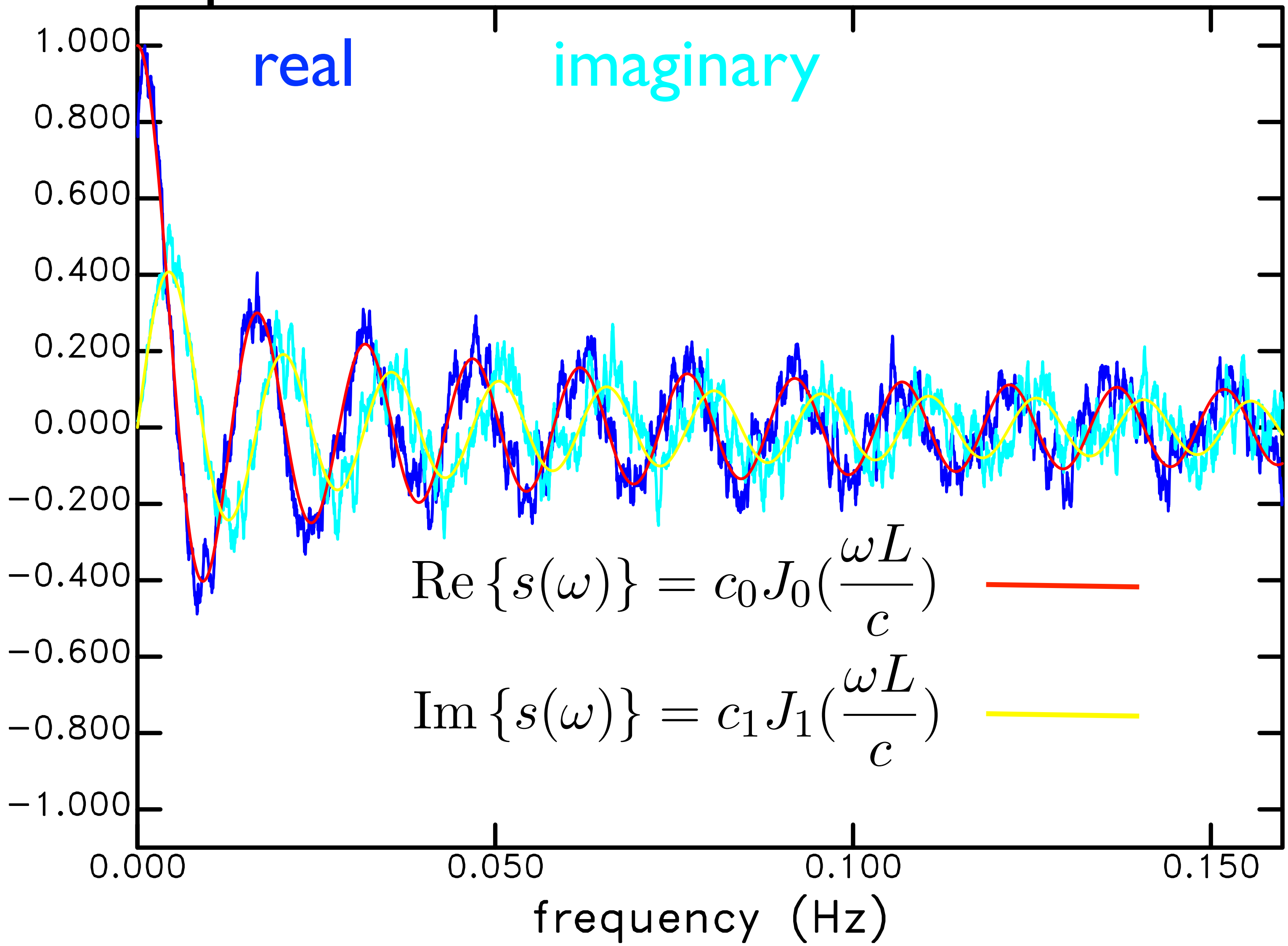



180 noise sources
azimuth 0 deg.
P
Q
azimuthally
varying amplitudes

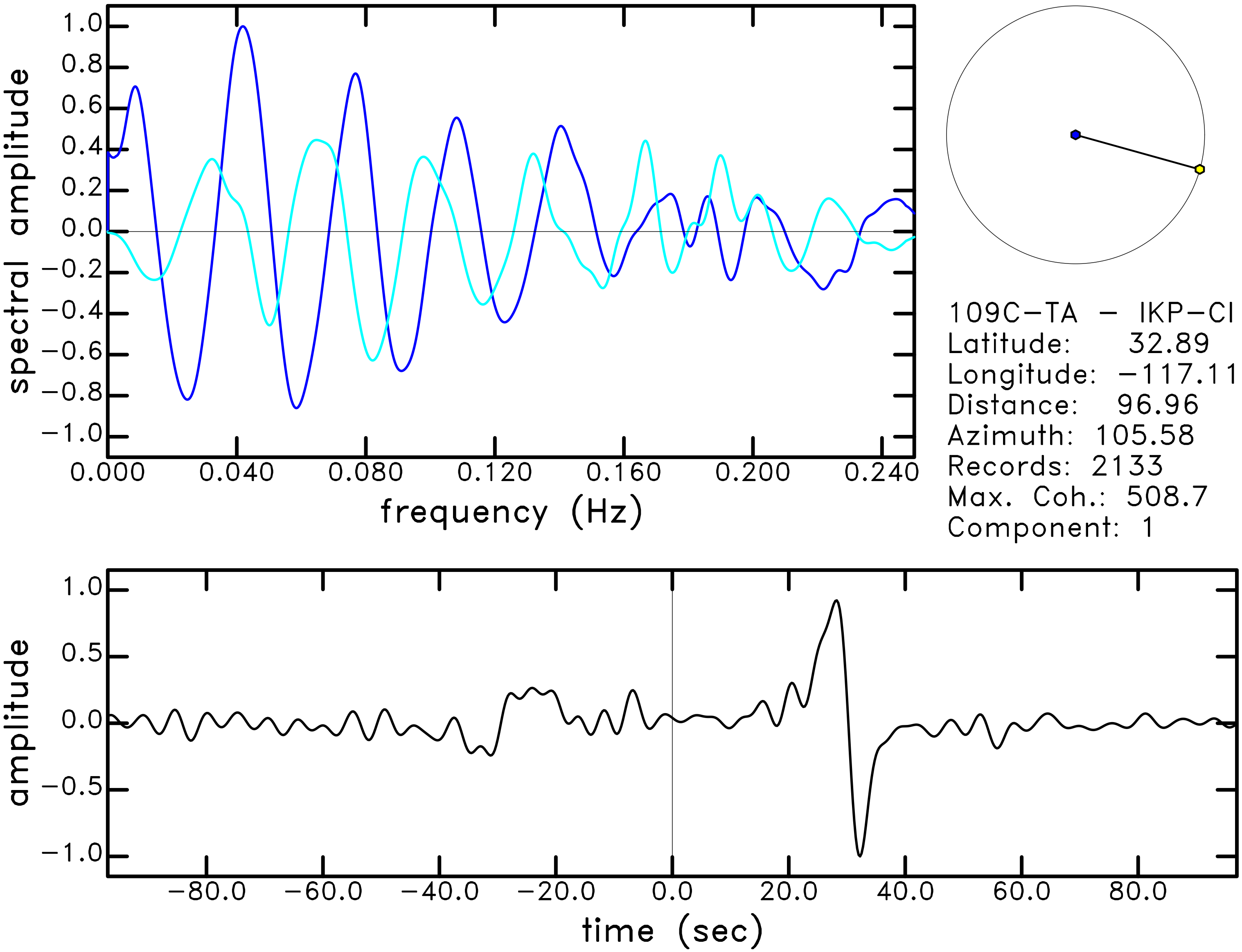
Cross-correlation function, P and Q



Spectrum of cross-correlation function

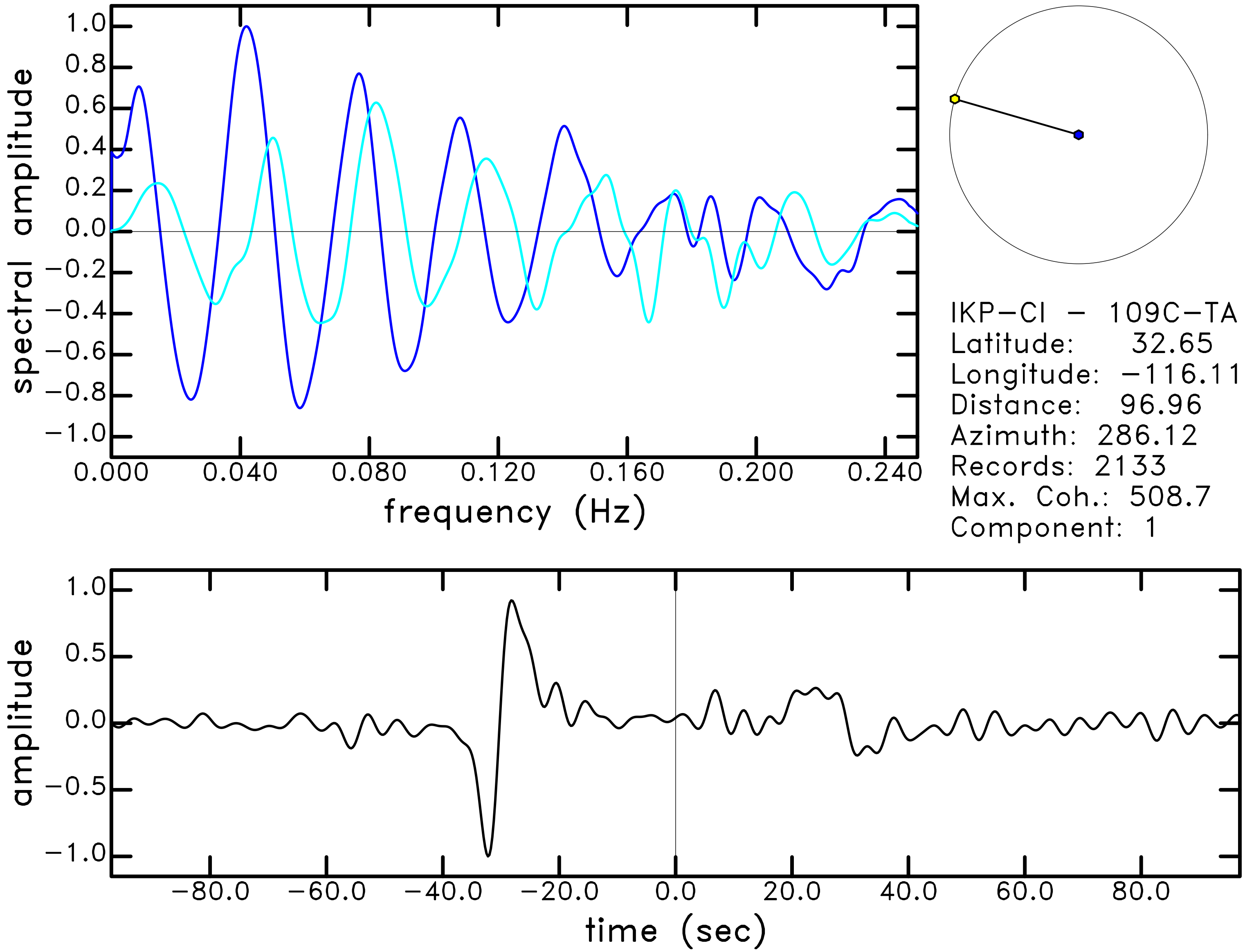


Real data

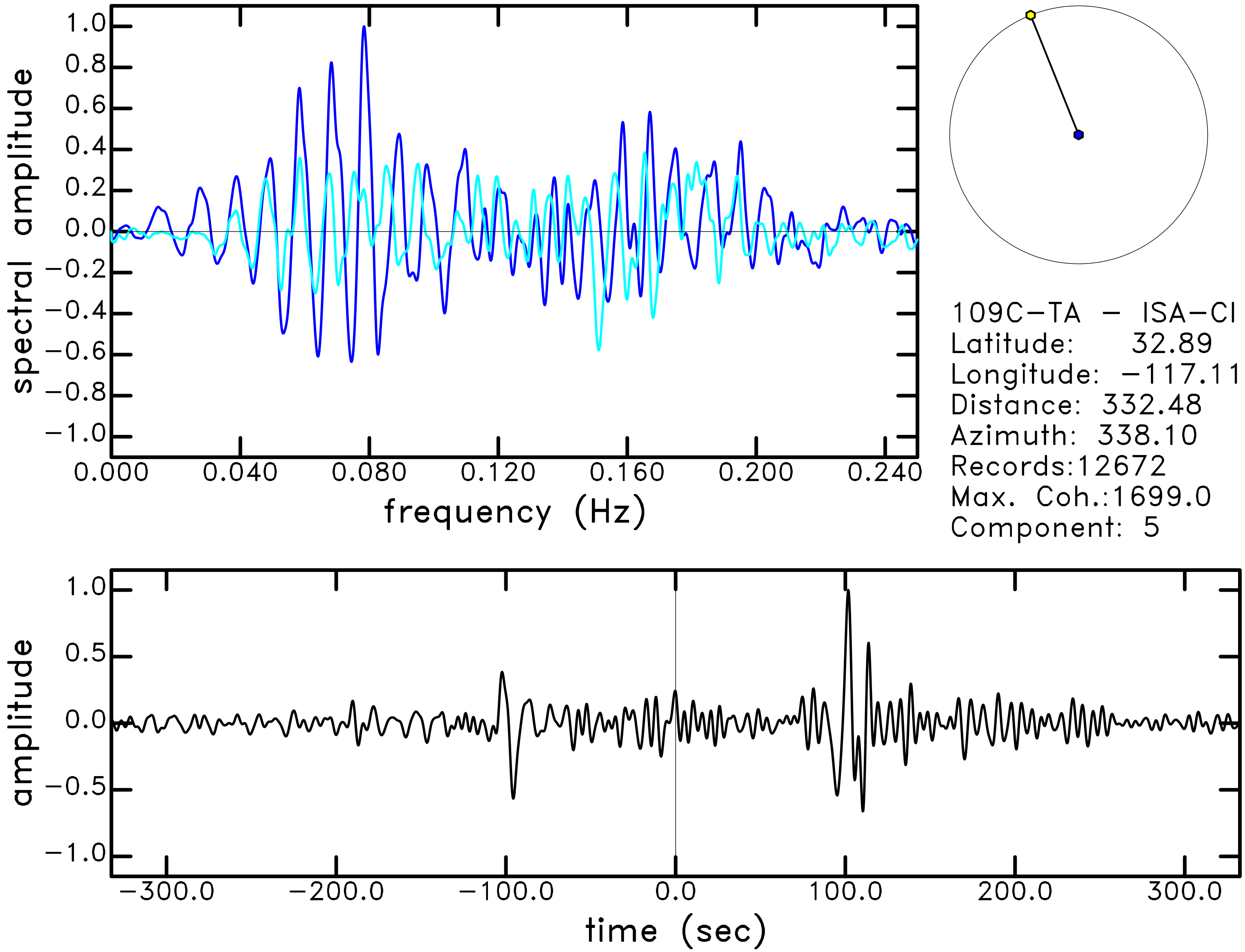


spectral amplitude
frequency (Hz)
109C–TA – IKP–CI
Latitude: 32.89
Longitude: –117.11
Distance: 96.96
Azimuth: 105.58
Records: 2133
Max. Coh.: 508.7
Component: 1
amplitude
time (sec)

Real data, reversed

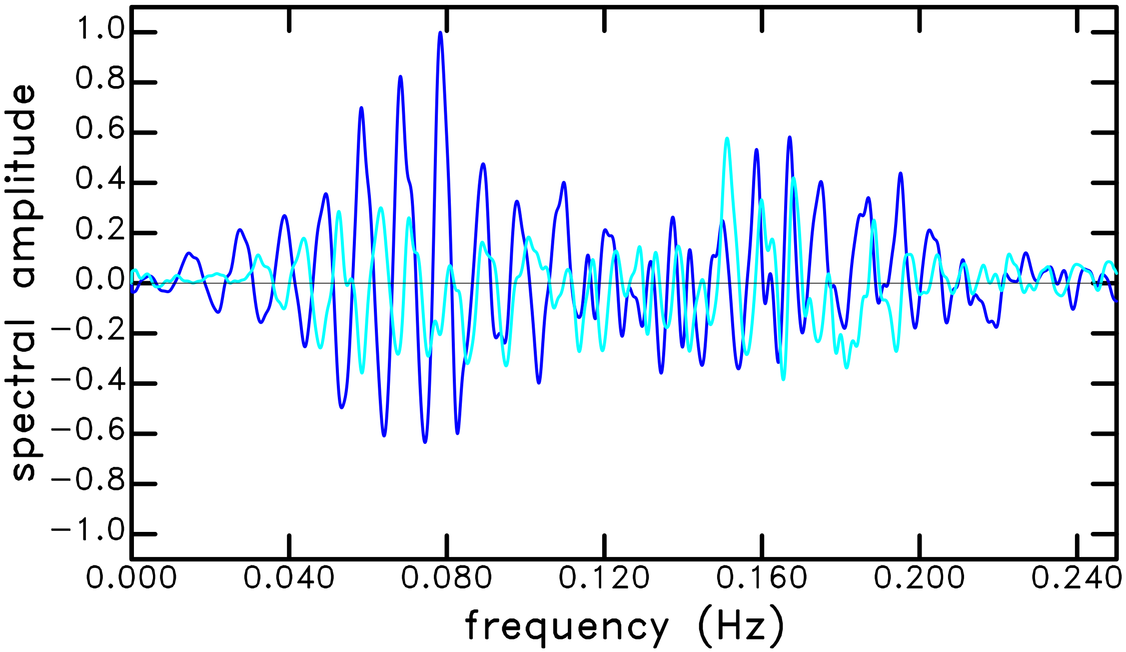


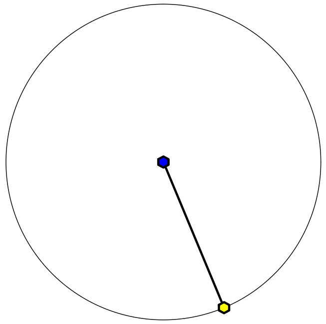
Love waves



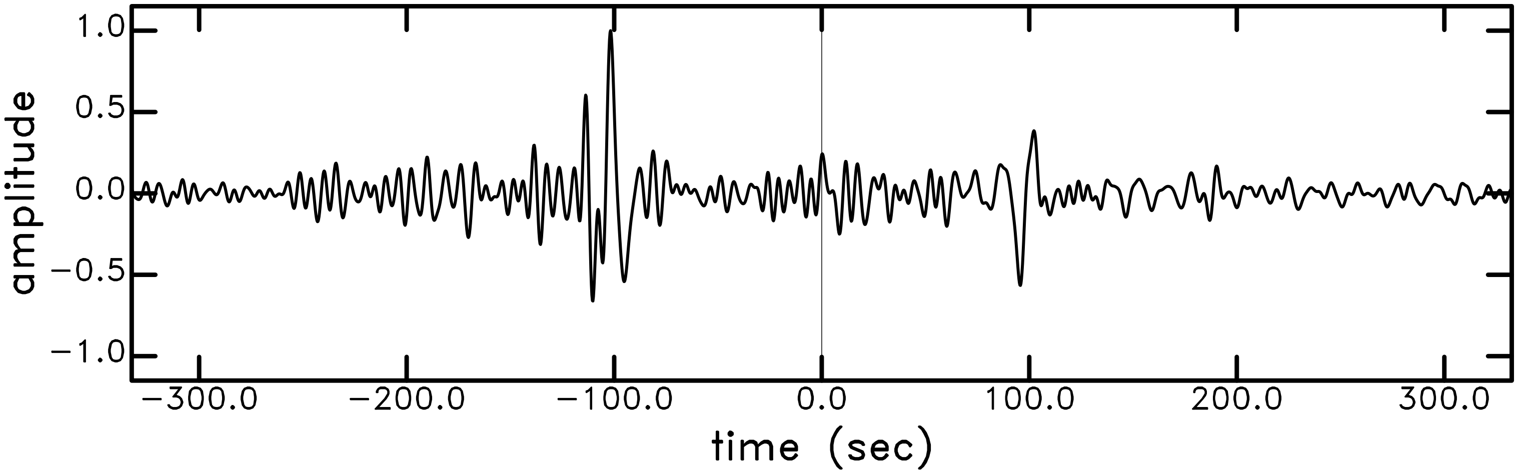
spectral amplitude
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
0.000
0.040
0.080
0.120
0.160
0.200
0.240
frequency (Hz)
109C−TA − ISA−CI
Latitude: 32.89
Longitude: −117.11
Distance: 332.48
Azimuth: 338.10
Records:12672
Max. Coh.:1699.0
Component: 5
amplitude
1.0
0.5
0.0
−0.5
−1.0
−300.0
−200.0
−100.0
0.0
100.0
200.0
300.0
time (sec)

Love waves, reversed





ISA−CI − 109C−TA
Latitude: 35.66
Longitude: −118.47
Distance: 332.48
Azimuth: 157.33
Records:12672
Max. Coh.:1699.0
Component: 5



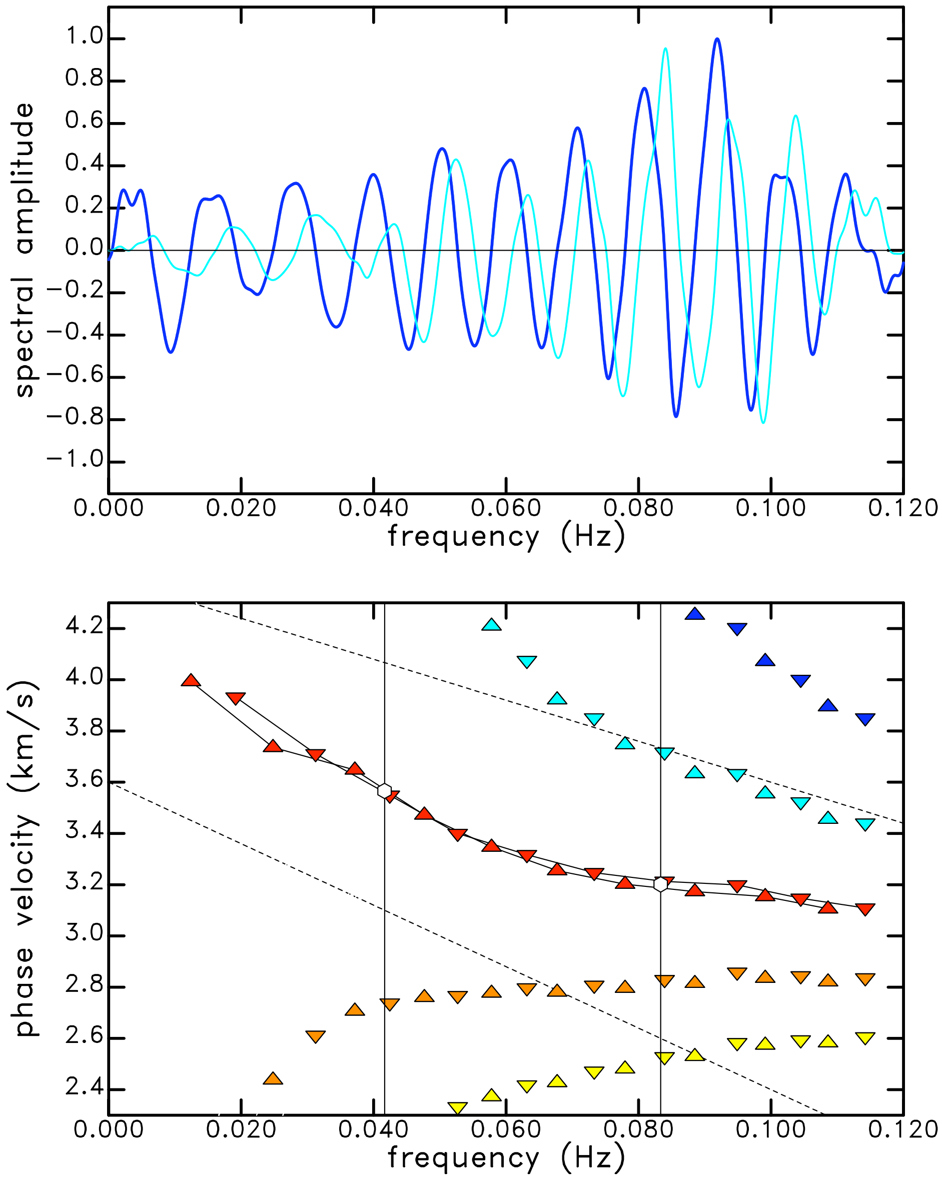
$$\overline{\rho}(r, \omega_0) = J_0\left(\frac{\omega_0}{c(\omega_0)} r\right)$$

"This formula clearly indicates that if one measures $\overline{\rho}(r, \omega_0)$ for a certain r and for various ω_0's, he can obtain the function $c(\omega_0)$, i.e., the dispersion curve of the wave for the corresponding range of frequency ω_0".

Aki, 1957

(made fashionable again by Ekström, Abers, and Webb, 2009)

Matching zero crossings for dispersion

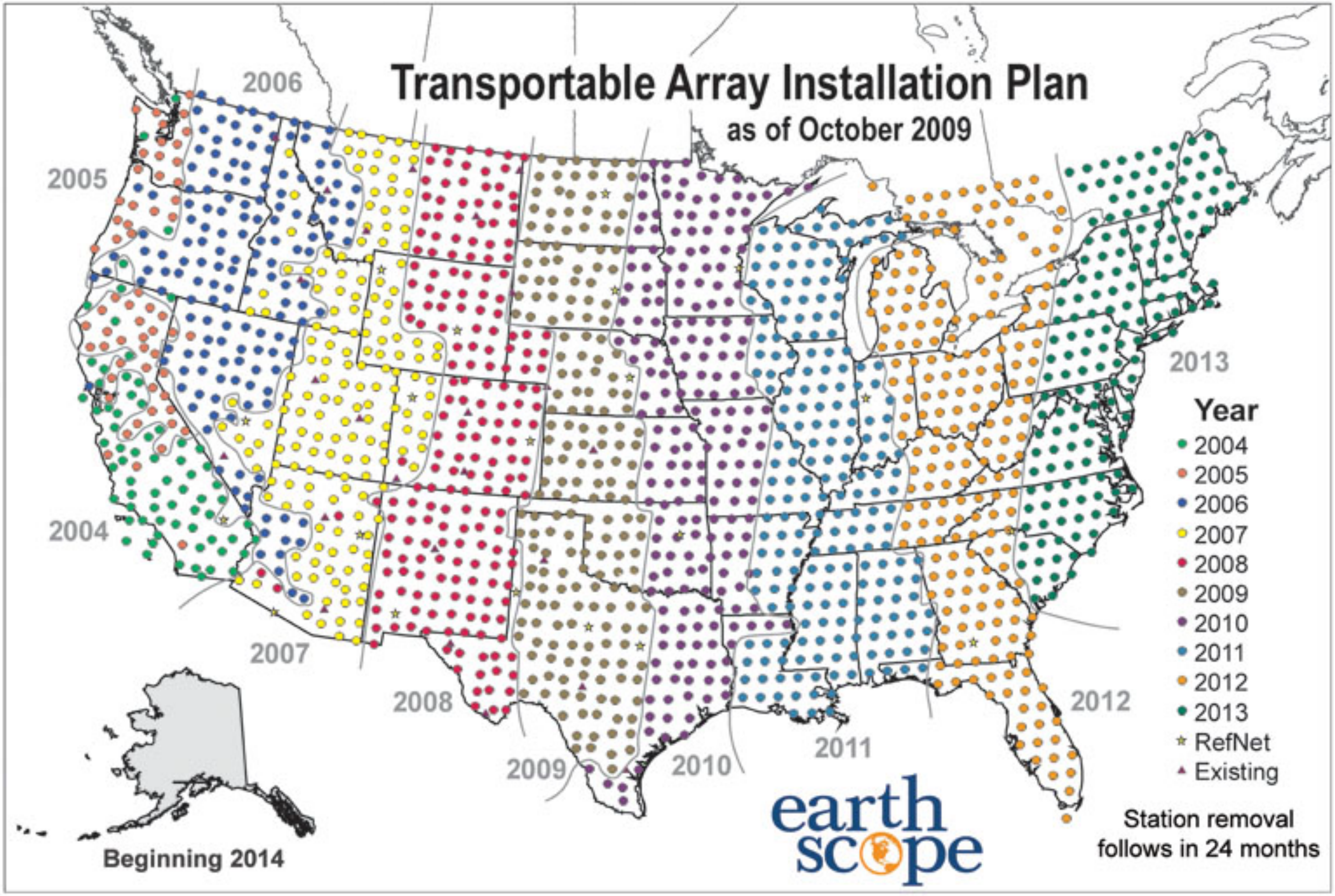


D07A-B04A

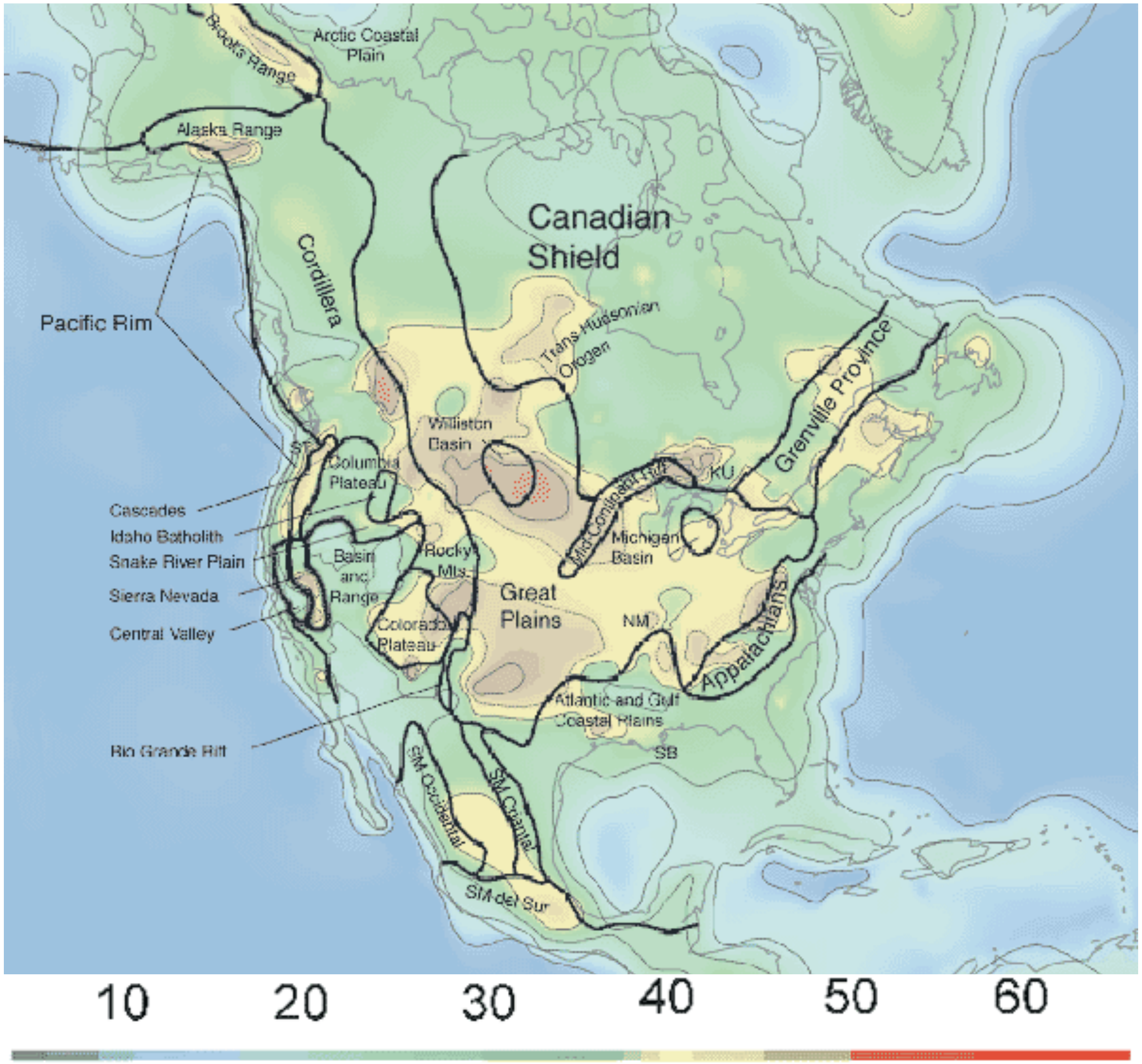
282 km

$$c(\omega_n) = \frac{\omega_n r}{z_n}$$

Some results from processing the USArray data 200601-201403


Transportable Array Installation Plan
as of October 2009
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
Year
2004
2005
2006
2007
2008
2009
2010
2011
2012
2013
RefNet
Existing
Beginning 2014
earthscope
Station removal
follows in 24 months

The crust of North America



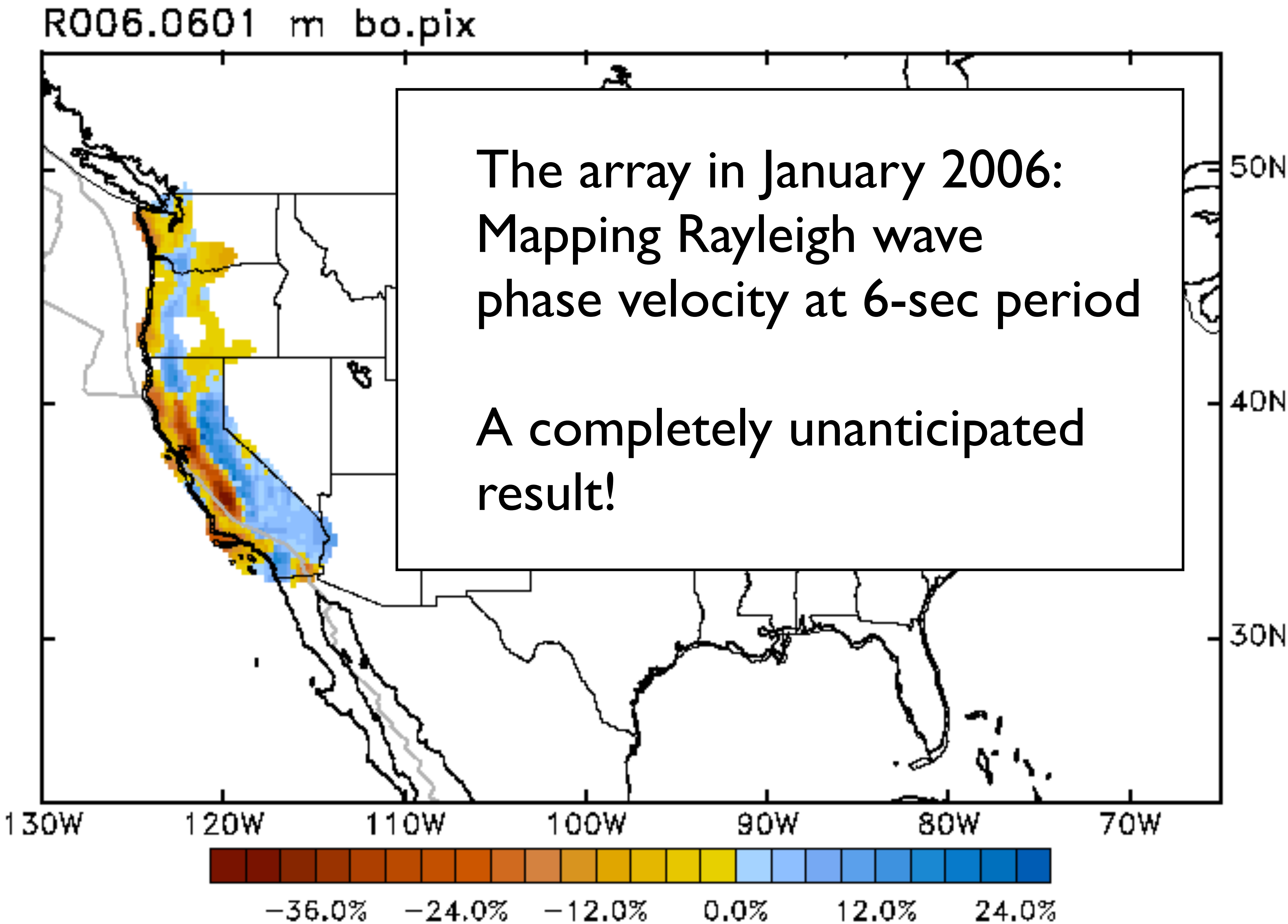
Chulick and Mooney, 2002

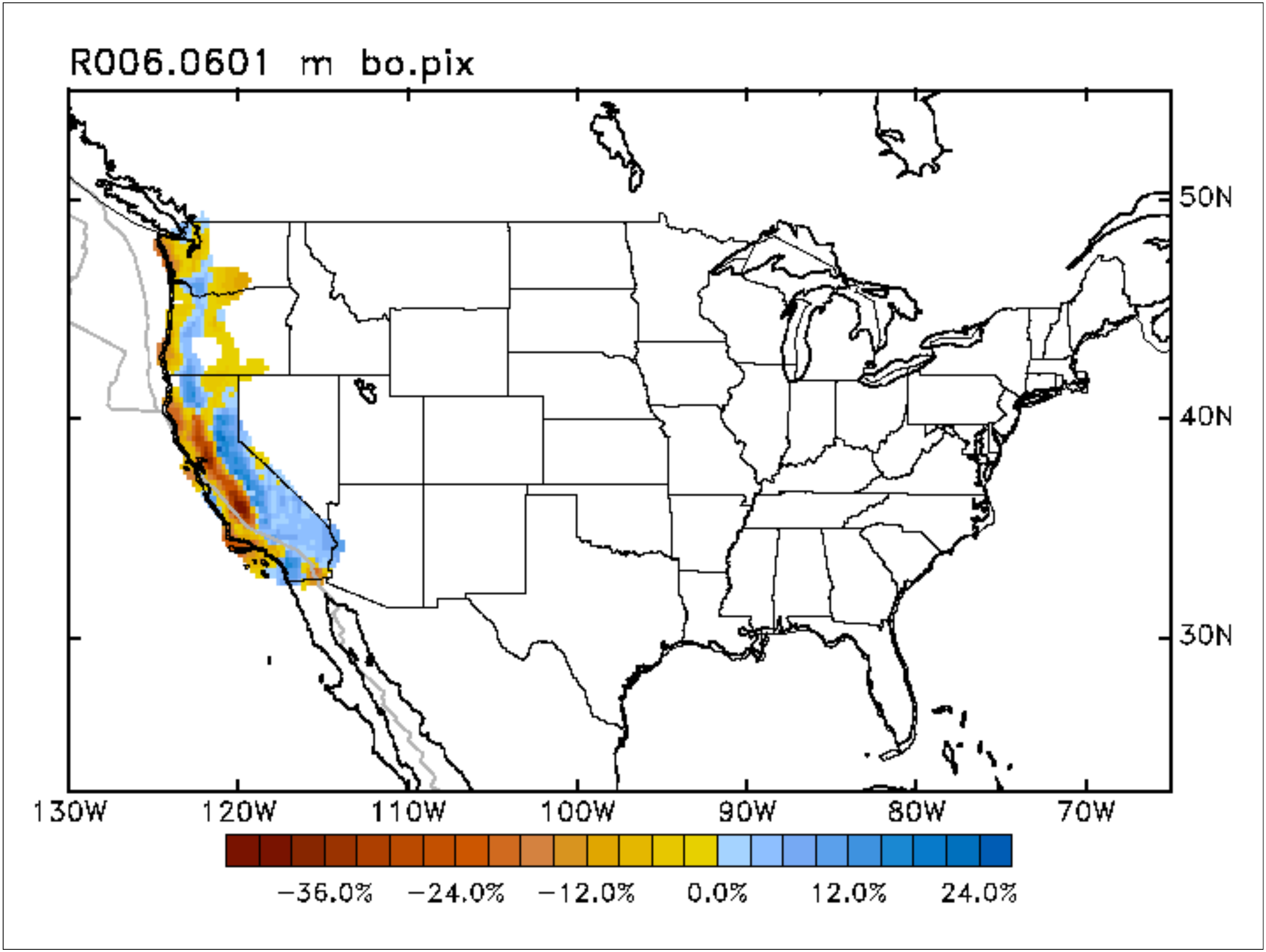
Recipe for success:

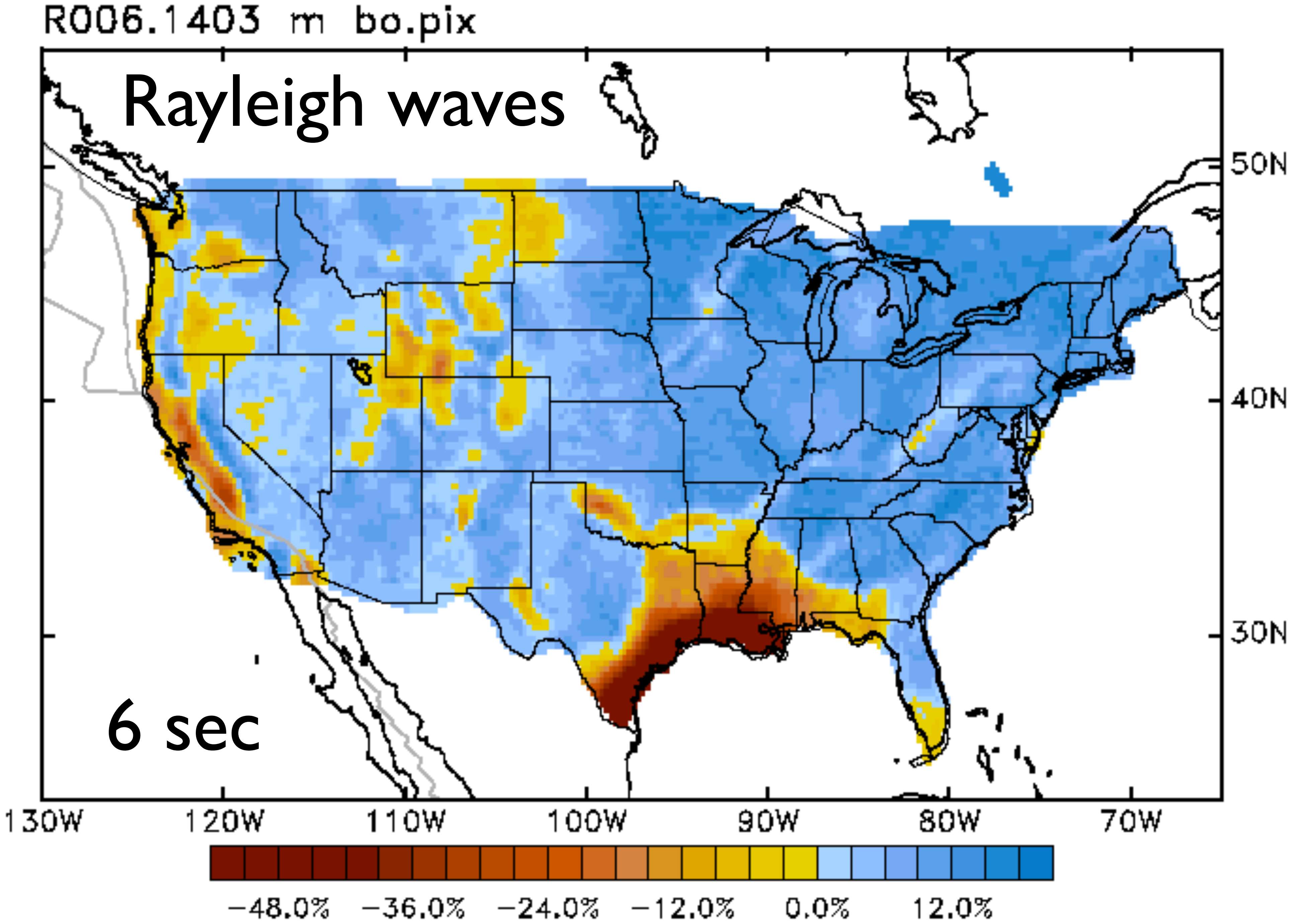
1. Correlate continuous recorded signals at all pairs of USArray stations in 4-h windows (note - this is a big calculation)
2. Stack all correlation functions for each pair
3. Determine zero crossings of stacked cross-correlation spectra
4. Determine phase velocities using Aki's formula
5. Invert phase-velocity observations to determine phase-velocity maps

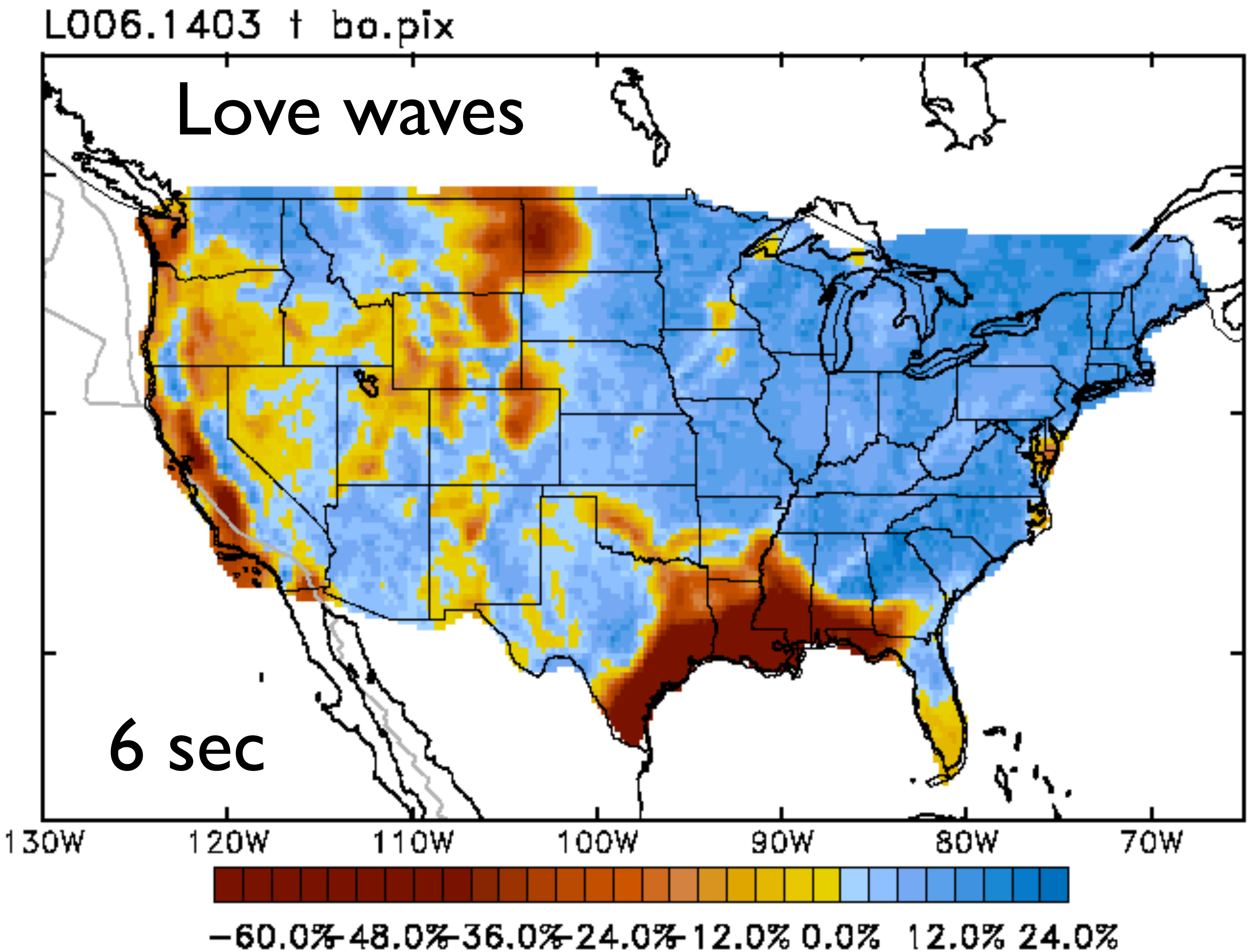
The array in January 2006:
Mapping Rayleigh wave phase velocity at 6-sec period

A completely unanticipated result!

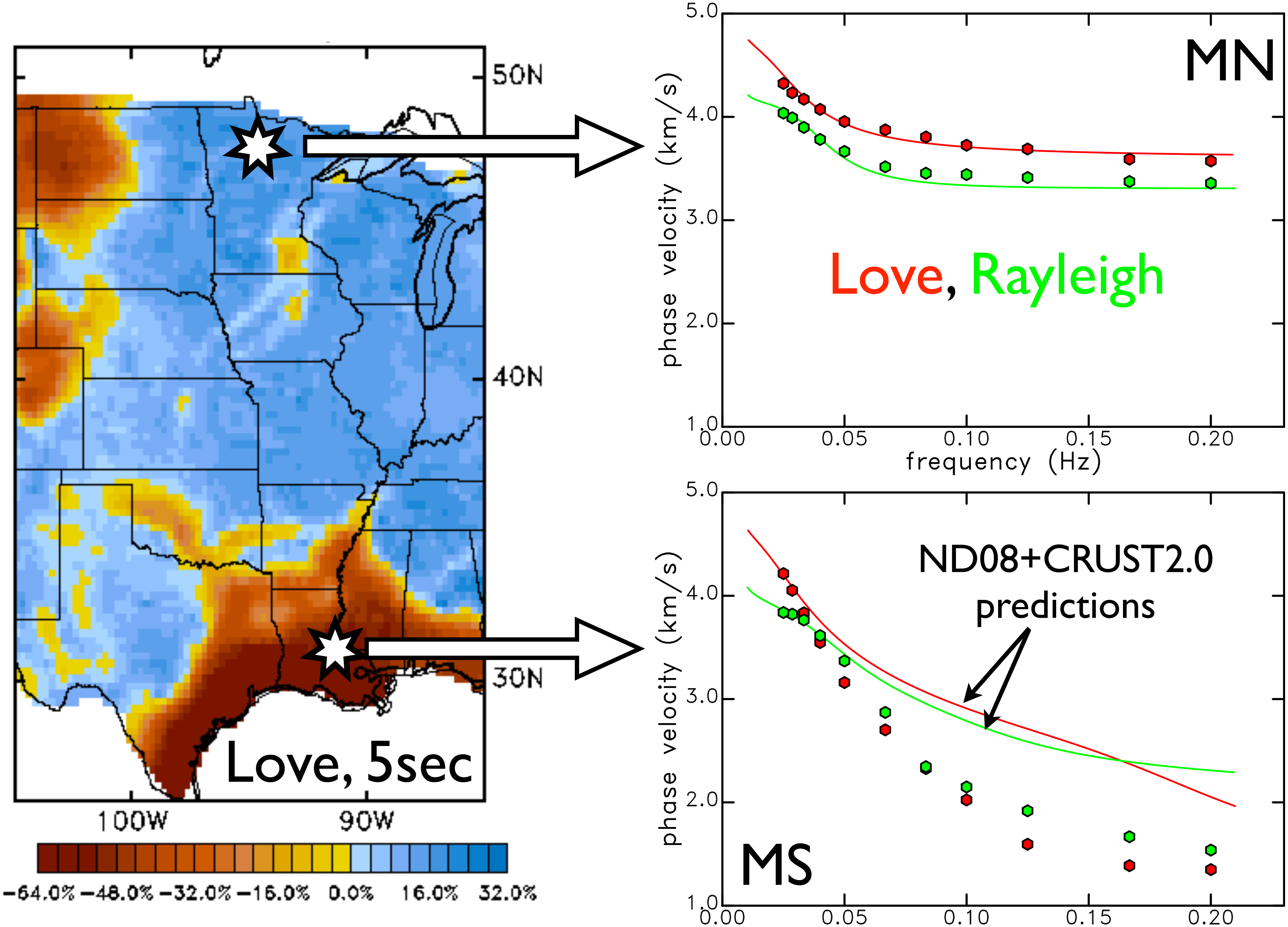



R006.0601 m bo.pix
50N
40N
30N
130W
120W
110W
100W
90W
80W
70W
−36.0%
−24.0%
−12.0%
0.0%
12.0%
24.0%


R006.1403 m bo.pix
Rayleigh waves
6 sec
50N
40N
30N
130W
120W
110W
100W
90W
80W
70W
−48.0%
−36.0%
−24.0%
−12.0%
0.0%
12.0%


L006.1403 t bo.pix
Love waves
6 sec
50N
40N
30N
130W
120W
110W
100W
90W
80W
70W
−60.0% −48.0% −36.0% −24.0% −12.0% 0.0% 12.0% 24.0%

Observed and predicted dispersion



1. Noise tomography is a powerful tool to investigate shallow Earth structure using data from a regional network

2. There are different algorithms that are used -- Aki's method is perhaps the simplest

3. Noise tomography requires continuous data